Introduction to Physics
CALCULATIONS

by

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CHAPTER 1: Units in Physics

UNITS

Measurements in Physics should use the *INTERNATIONAL SYSTEM OF UNITS (SI).*

This system develops a whole range of units (DERIVED UNITS) from the following BASE UNITS:

QUANTITY	BASE SI UNIT	SYMBOL
length	metre	m
mass	kilogram	kg
time	second	s
temperature	kelvin	K
amount of substance	mole	mol
electric current	ampere	A

Some commonly used DERIVED UNITS are:

QUANTITY	DERIVED SI UNIT	SYMBOL
area	square metres	m^2
volume	cubic metres	m^3
density	kilograms per cubic metre	$kg\ m^{-3}$ (or kg/m^3)
velocity (speed)	metres per second	$m\ s^{-1}$ (or m/s)
acceleration	metres per second squared	$m\ s^{-2}$ (or m/s^2)
force	kilograms metres per second squared	$kg\ m\ s^{-2}$ ($kg\ m/s^2$)

NOTE: The newton (N) is the unit used to measure force. One newton equals one $kg\ m\ s^{-2}$.

Commonly used prefixes:

PREFIX	SYMBOL	MEANING
deci	d	1/10 (10^{-1})
centi	c	1/100 (10^{-2})
milli	m	1/1 000 (10^{-3})
micro	μ	1/1 000 000 (10^{-6})
nano	n	1/1 000 000 000 (10^{-9})
kilo	k	1 000 (10^3)
mega	M	1 000 000 (10^6)
giga	G	1 000 000 000 (10^9)

CONVERSION

Although a range of units are used it is essential that in Physics, base or derived units be used.

TYPE EXAMPLE 1: Determine the number of metres in 2.5 km.

$$2.5\ km = (2.5 \times 1\ 000)\ m$$
$$= 2\ 500\ m\ (\text{or } 2.5 \times 10^3\ m)$$

TYPE EXAMPLE 2: How many seconds are contained in 1.2 hours?

$$1.2 \text{ hours} = (1.2 \times 60 \times 60)\text{ s}$$
$$= 4\,320 \text{ s (or } 4.32 \times 10^3 \text{ s)}$$

TYPE EXAMPLE 3: Calculate the number of metres in 7 cm.

$$7 \text{ cm} = \frac{7}{100} \text{ m}$$
$$= 0.07 \text{ m (or } 7 \times 10^{-2} \text{ m)}$$

SET 1: CONVERSION

Find the number of:

1. metres in 6.2 km
2. seconds in 3.1 minutes
3. seconds in 2.5 hours
4. metres in 176 cm
5. kilograms in 5 500 grams
6. kilograms in 25 grams
7. kilograms in 1.5 tonnes
8. metres in 18 mm
9. square metres (m^2) in 750 square centimetres (cm^2)
10. cubic metres (m^3) in 1 000 cubic centimetres (cm^3)

CHAPTER 2: Motion

MOTION

Motion may be defined as the continuous change in position of an object in relation to another object which is regarded as being at rest.

SCALAR AND VECTOR QUANTITIES

Physical quantities which can be accurately described by the use of a number with the appropriate units are called *SCALAR QUANTITIES (OR SCALARS).*

Scalar quantities have *MAGNITUDE* but *NO DIRECTION.*

Other physical quantities cannot be accurately described in terms of magnitude only. These quantities, which require a direction to be stated, in addition to the magnitude, for an accurate description are called *VECTOR QUANTITIES (OR VECTORS).*

Vector quantities have *MAGNITUDE* and *DIRECTION.*

Examples of scalar and vector quantities:

SCALAR	VECTOR
age (e.g. 25 years)	force (e.g. 5 N south)
time (e.g. 30 s)	displacement (e.g. 52 m south-west)
area (e.g. 8.4 m^2)	velocity (e.g. 4 m s^{-1} east)
distance (e.g. 7.3 m)	acceleration (e.g. 10 m s^{-2} downwards)
mass (e.g. 46 kg)	
speed (e.g. 5 m s^{-1})	
population (e.g. 15 000 000 people)	

REPRESENTATION OF VECTORS

Vector quantities are conveniently represented by arrows. The length of the arrow represents the magnitude of the vector quantity and the direction of the arrow indicates the direction of the vector.

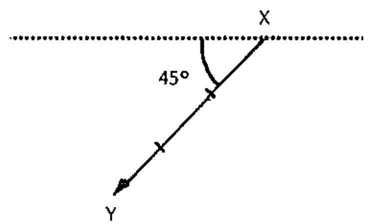
For all graphical representation of vectors clearly indicate the:

1. reference direction (e.g. north).
2. scale (e.g. 1 cm = 10 cm).

TYPE EXAMPLE 4: Show graphically a displacement of 30 m south-west.

Direction: 

Scale: 1 cm = 10 m



XY represents a displacement of 30 m south-west

SET 2: GRAPHICAL REPRESENTATION OF VECTORS

Show graphically each of the following vector quantities:

1. A displacement of 400 m due south.
2. A velocity of 7 m s^{-1} W 30° N.
3. A force of 80 N at 45° to the horizontal.
4. An acceleration of 2 m s^{-2} to the left.
5. A displacement of 24 km north-west.

DISPLACEMENT

Displacement is defined as the straight line distance between the starting and finishing points.

Consider the travel path shown below (..............) between two towns, A and B.

The ....path represents the total *DISTANCE* travelled. The straight line AB, the direct distance between A and B, is the *DISPLACEMENT* in the direction of the line AB.

As displacement is distance in a given direction, it is a vector quantity.

UNIT OF DISTANCE AND DISPLACEMENT.

The unit is that of length, the metre (m).

RESULTANT OF TWO (OR MORE) DISPLACEMENTS

When a body is subjected to two (or more) displacements the final direct distance (the displacement) from the starting position is *NOT* equal to the total distance travelled (unless the individual displacements are in the same direction).

To determine the exact location (distance and direction from the starting position) of a body, it is necessary to determine the *RESULTANT* displacement.

A resultant of two (or more) vectors may be defined as the single vector which on its own will produce the same effect as the two (or more) vectors combined.

DETERMINATION OF RESULTANTS

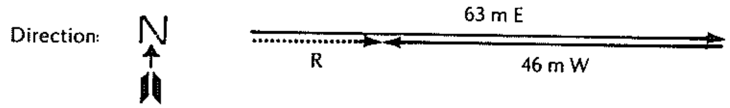
To determine the resultant:

1. Place the vectors head to tail (in any order).
2. Draw in the resultant which is from the tail of the first vector to the head of the last.
3. If the vectors have been drawn to scale then the magnitude of the resultant and the direction can be measured — otherwise both can be calculated.

This method is often referred to as the *"head to tail"* or *"triangle"* method.

TYPE EXAMPLE 5: What is the resultant displacement of a displacement of 63 m east and a displacement of 46 m west?

Sketch vectors head to tail (approximately to scale) and the resultant displacement.



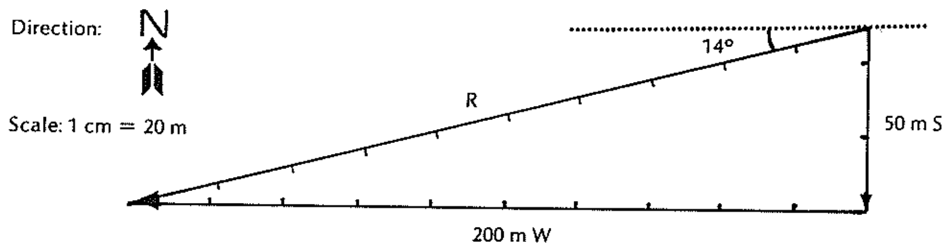
For displacements in one straight line, measurement (i.e. graphical solution) is unnecessary as the calculation of the magnitude of the resultant is obvious.

$$\text{resultant, } R = (63 \text{ m}) - (46 \text{ m}) = 17 \text{ m}$$

i.e. the resultant displacement is 17 m east.

TYPE EXAMPLE 6: Determine the resultant displacement if a person walks 50 m south to a corner and then 200 m west to go to the local store.

A. GRAPHICAL SOLUTION



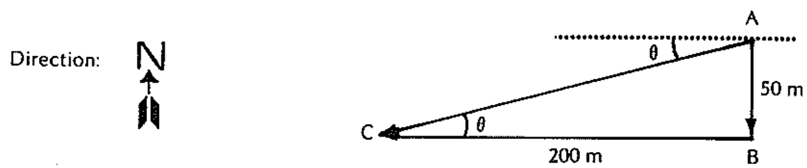
$$\text{length of } R = 10.3 \text{ cm}$$

$$\therefore \text{ magnitude of } R = (10.3 \times 20) \text{ m} = 206 \text{ m}$$

i.e. the resultant displacement is 206 m W 14° S

B. CALCULATION SOLUTION (right angled triangles only).

1. Sketch the vector triangle (approximately to scale).



2. Using Pythagorus' Theorem calculate a value for the magnitude of the resultant, AC.

AC = resultant
= ?
AB = 50 m S
BC = 200 m W

$$AC^2 = AB^2 + BC^2$$
$$= (50)^2 + (200)^2$$
$$= 2\,500 + 40\,000$$
$$= 42\,500$$
$$\therefore AC = 206.2 \text{ m}$$

3. Using trigonometric ratios determine the direction of the resultant.

$$\tan\theta = \frac{AB}{BC} = \frac{50}{200} = 0.25$$
$$\therefore \theta = 14°$$

i.e. the resultant displacement is 206.2 m W 14° S.

SET 3: RESULTANT DISPLACEMENT I

Determine the resultant displacement of:

1. A run of 52 m due south followed by a run of 164 m due south.
2. A football kicked 35 m directly down field then punched a further 12 m in the same direction.
3. 65 m north, 78 m south.
4. A drive of 164 km SW, then 49 km NE.
5. A ball dropped from a building 25 m high onto the ground, bouncing vertically upwards and caught 2 m above the ground.

SET 4: RESULTANT DISPLACEMENT II

Determine graphically the resultant displacement for each of the following:

1. A boy walks 200 m due south, then 100 m south-west.
2. A plane flies 250 km W then 500 km S.
3. 40 cm to the left, 20 cm vertically upwards.
4. A ball rolling 50 cm down an incline (30° to the horizontal) and then 30 cm along the horizontal.
5. 14 m N, 10 m W, 5 m SW.

SET 5: RESULTANT DISPLACEMENT III

Using Pythagorus' Theorem and trigonometric ratios determine the resultant displacement for each of the following:

1. 300 km N, 300 km W.
2. A climb 6 m out of a well and 8 m away from it.
3. 500 m E, 1 200 m S.
4. 17.32 m horizontally then 10 m vertically upwards.
5. 1.2 km NW, 1.6 km NE.

SPEED

The speed of a body is the rate of change of distance by the body.

The average speed of a body over a distance travelled in a time interval can be determined by:

$$\boxed{\text{Speed (av)} = \frac{d}{t}}$$

where d = distance (measured from zero)
t = time interval (measured from zero)

Speed is a scalar quantity as it involves no specific direction. Distance is the total length of the path covered by the body.

If to accurately describe motion direction is required, then *VELOCITY* is used in place of speed.

VELOCITY

The velocity of a body is the rate of change of displacement of the body.

The average velocity of a body over a displacement in a measured time interval can be determined by:

$$\boxed{v_{av} = \frac{s}{t}}$$

v_{av} = average velocity
s = displacement (measured from zero)
t = time interval (measured from zero)

Velocity is a vector quantity. *The direction of the velocity is that of the displacement (unless otherwise stated).*

UNITS OF VELOCITY AND SPEED

Velocity and speed are both determined by dividing units of length (m) by units of time (s).

Units: $\frac{\text{metres}}{\text{seconds}} = \frac{m}{s} = m\,s^{-1}$ (or m/s)

DETERMINATION OF DISPLACEMENT

The equation for average velocity can be rearranged to determine the displacement.

$$v_{av} = \frac{s}{t}$$

rearranging gives: $s = v_{av} \times t$
similarly $d = \text{speed (av)} \times t$

TYPE EXAMPLE 7: Calculate the average velocity of a vehicle which travels 3 km east in 5 minutes.

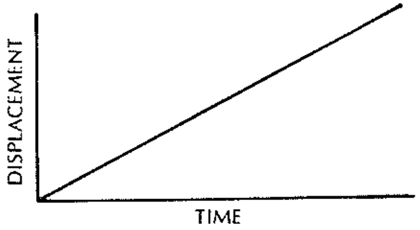
v_{av} = ?
s = 3 km
= (3 × 1 000) m
t = 5 minutes
= (5 × 60) s

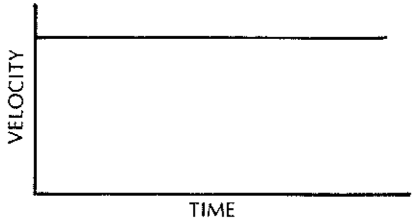
$$v_{av} = \frac{s}{t} = \frac{3 \times 1\,000}{5 \times 60} = 10\ m\,s^{-1}$$

i.e. the average velocity is 10 m s^{-1} east.

NOTE: 1. While speed and direction remain unchanged the velocity remains the same.

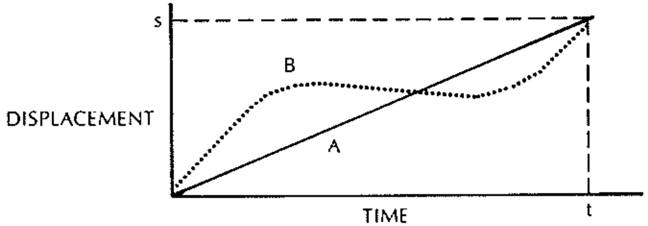
2. The graphs below represent motion with a velocity which does not vary during the considered period of time.





Velocity which does not vary is termed CONSTANT or UNIFORM VELOCITY.

3. Consider the two motions represented below.



Motions A and B both represent a displacement s in a time interval t.

∴ in each case v_{av} is the same (s/t)

Motion A is uniform (or constant) velocity while motion B is a varying velocity. Therefore for motion A, the velocity at any instant is equal to the average velocity which is equal to the uniform (or constant) velocity.

TYPE EXAMPLE 8: Determine the displacement of a body which moves with a uniform velocity of 2.4 m s⁻¹ for 50 s.

$v_{av} = 2.4 \text{ m s}^{-1}$
$s = ?$
$t = 50 \text{ s}$

$$v_{av} = \frac{s}{t}$$

$$\therefore \quad s = v_{av} \times t = 2.4 \times 50 = 120 \text{ m}$$

i.e. the displacement is 120 m in the direction of the velocity.

SET 6: VELOCITY I

1. What is the average speed of a sprinter who runs 100 m in 12.5 s?
2. A car takes 80 s to complete one lap of a 2 km circuit. What is its average speed (in m s^{-1})?
3. A body travels 36 m in 4 s. What is its average speed?
4. How far will a bird fly in 25 s travelling with an average speed of 6 m s^{-1}?
5. A person swims from one end of a pool to the other at an average velocity of 1.5 m s^{-1} in 20 s. What is the length of the pool?
6. Determine the distance travelled by a plane in 1 hour if it flies due south with a uniform velocity of 90 m s^{-1}
7. How long will it take for a body moving with a uniform velocity of 7 m s^{-1} to cover 98 m?
8. A cricket ball is thrown a distance of 90 m from the boundary to the wicket at an average horizontal speed of 30 m s^{-1}. How long will it take for the ball to reach the wicket?
9. Determine the time taken for a horse running with an average speed of 14 m s^{-1} to complete one circuit of a 840 m track.
10. How long will it take to complete a trip from Perth to Kalgoorlie, a distance of 595 km, travelling at an average speed of 70 km h^{-1}?
11. A train moving with an average speed of 80 km h^{-1} takes 0.25 hours to travel from one railway siding to the next. What is the distance between the two sidings?
12. Determine the average speed (km h^{-1}) of a plane which flies a distance of 720 km in 1.5 hours?

SET 7: VELOCITY II

1. An athlete, commencing at a point due east, completes half a lap of a 440 m track in 25 s. Determine the athlete's:
   (a) average speed.
   (b) average velocity (in westerly direction).
   (c) average velocity if the athlete completes one full lap at the same pace.
2. A car travels with an average speed of 20 m s^{-1} due north for 3 km then due west for another 4 km calculate:
   (a) the total time taken for the journey.
   (b) the displacement of the car.
   (c) the average velocity of the car.
3. A person walks for 10 s at 1 m s^{-1} then for another 10 s in the same direction at 2 m s^{-1}. Find the:
   (a) total displacement.
   (b) average velocity.
4. A body travels 200 m at a uniform velocity of 4 m s^{-1} and a further 150 m in the same direction at a uniform velocity of 7.5 m s^{-1}. Determine the:
   (a) total time taken.
   (b) average velocity for the whole journey.
5. A cyclist travels at 6 m s^{-1} S for 10 minutes and then returns 600 m along the same path. What is the final displacement of the cyclist?

6. In a 50 m swimming race the winner completed the race in 32 seconds. By what margin (length) did the winner defeat the second placegetter who swam with an average velocity of 1.5 m s^{-1}?
7. A cyclist moving uniformly at 5 m s^{-1} overtakes a pedestrian walking in the same direction in 6 s. If the pedestrian initially was 18 m ahead of the cyclist, find the pedestrian's average velocity.
8. Determine the time saved by travelling 8 km at 40 m s^{-1} rather than at 32 m s^{-1}.
9. How far will a car moving at 20 m s^{-1} travel in the same time that a car moving at 25 m s^{-1} travels 1 km?
10. A car travelling at 15 m s^{-1} is 10 m behind another vehicle moving in the same direction. If it takes 10 s to overtake this vehicle, what is the speed of the slower vehicle?

ACCELERATION

Acceleration occurs whenever the velocity of a body changes.

Acceleration is defined as the rate of change of velocity.

For the purpose of examples considered in this text acceleration is considered to be uniform (or constant).

From the definition:

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$= \frac{\text{final velocity} - \text{initial velocity}}{\text{time}}$$

$$a(av) = \frac{v - u}{t}$$

rearranging gives: $\boxed{v = u + at}$ (assuming uniform acceleration)

where: v = final velocity
u = initial velocity
a = uniform acceleration
t = time interval (measured from zero)

UNITS OF ACCELERATION

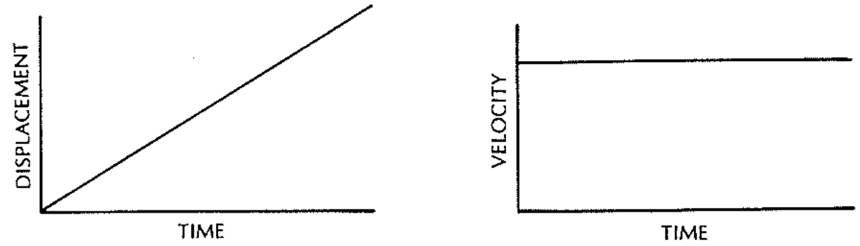
As acceleration is the rate of change of velocity then its units are those of velocity divided by time.

$$\text{Units: } \frac{\text{m s}^{-1}}{\text{s}} = \text{m s}^{-2} \text{ (or m/s}^2\text{)}$$

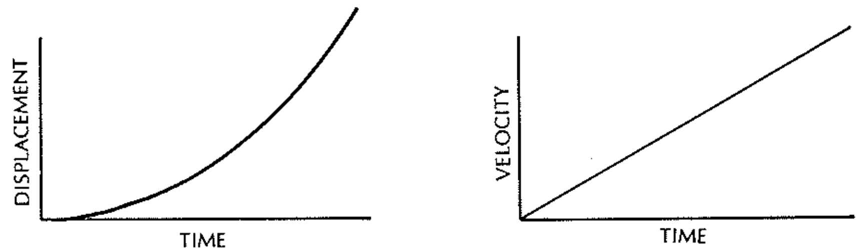
GRAPHICAL REPRESENTATION OF ACCELERATED MOTION

Consider the following graphical representations of motion:

(a) Uniform velocity (zero acceleration)



(b) Uniform acceleration



STRAIGHT LINE MOTION

If the motion being considered is in one straight line, then vectors (i.e. displacement, acceleration and velocity) can only have one of two directions.

The following sign convention for vector directions will be used:

1. *Consider the direction of initial motion to be positive. All vectors in this direction will be given a positive value.*
2. *Assume any stated acceleration to be in the direction of the initial motion and hence positive, unless stated otherwise.*
3. *If an acceleration is found to be*
   (a) positive, then it is in the direction of the initial motion and velocity increases uniformly.
   (b) negative, then it is in the opposite direction to the initial motion and velocity decreases uniformly.

NOTE: For a body starting from rest, the initial velocity, u = 0.

TYPE EXAMPLE 9: A car starting from rest is accelerated at 2 m s^{-2}. Calculate the velocity after 5 s.

u = 0
v = ?
a = 2 m s^{-2}
t = 5 s

$v = u + at$
$= 0 + (2 \times 5)$
$= 10$ m s^{-1}

i.e. the velocity is 10 m s^{-1} (in the direction of the acceleration).

TYPE EXAMPLE 10: Determine the acceleration of a body which is accelerated from 2 $m\ s^{-1}$ to 10 $m\ s^{-1}$ in 2.5 s.

$u = 2\ m\ s^{-1}$
$v = 10\ m\ s^{-1}$
$a = ?$
$t = 2.5\ s$

$$v = u + at$$
$$\therefore a = \frac{v - u}{t} = \frac{(10 - 2)}{2.5} = \frac{8}{2.5} = 3.2\ m\ s^{-2}$$

i.e. acceleration is 3.2 $m\ s^{-2}$ (in the direction of the initial motion).

TYPE EXAMPLE 11: What is the initial velocity of a body which attained a velocity of 16 $m\ s^{-1}$ when accelerated at 1.5 $m\ s^{-2}$ for 8 s?

$u = ?$
$v = 16\ m\ s^{-1}$
$a = 1.5\ m\ s^{-2}$
$t = 8\ s$

$$v = u + at$$
$$\therefore u = v - at = 16 - (1.5 \times 8) = 16 - 12 = 4\ m\ s^{-1}$$

i.e. the initial velocity is 4 $m\ s^{-1}$ (in the direction of the final velocity)

SET 8: ACCELERATION

1. What is the increase in velocity in each second of a body accelerated at 1.8 $m\ s^{-2}$?
2. Calculate the final velocity of a motor car initially travelling at 10 $m\ s^{-1}$ which is accelerated at 1.4 $m\ s^{-2}$ for 10 s.
3. A body starts from rest and is accelerated at 2.7 $m\ s^{-2}$. What is the velocity after 12 s?
4. A ball rolling down a slope from rest is accelerated at 3.5 $m\ s^{-2}$. What is the ball's velocity after 6 s?
5. Find the acceleration of a plane which increases its velocity from 100 $m\ s^{-1}$ to 200 $m\ s^{-1}$ in 40 s.
6. What is the average acceleration of a body which is accelerated from rest to 15 $m\ s^{-1}$ in 5 s?
7. Determine the average acceleration experienced by a body which increases its velocity from 7 $m\ s^{-1}$ to 34 $m\ s^{-1}$ in 9 s.
8. A vehicle starting from rest is accelerated at 2.5 $m\ s^{-2}$. Determine the time taken to reach a velocity of 5 $m\ s^{-1}$.
9. Find the time taken for a body travelling with a velocity of 15 $m\ s^{-1}$ to reach a velocity of 29 $m\ s^{-1}$ when accelerated at 0.7 $m\ s^{-2}$.
10. A car is accelerated from 10 $m\ s^{-1}$ to 25 $m\ s^{-1}$ in 2.5 s. What is the average acceleration?

NEGATIVE ACCELERATION

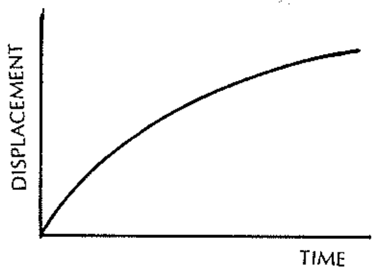
If a body (travelling in the positive direction) slows down uniformly it can be seen from the equation derived from the definition of acceleration that the value obtained for acceleration will be negative.

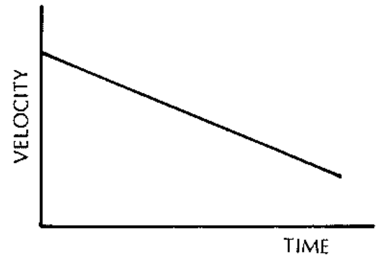
That is, as $a = \frac{v - u}{t}$ then if $v < u$, a will be negative.

Whenever the velocity vector and the acceleration vector are in opposite directions the magnitude of the velocity decreases uniformly.

A negative acceleration is also referred to as *RETARDATION* or *DECELERATION*.

GRAPHICAL REPRESENTATION OF UNIFORM NEGATIVE ACCELERATION





TYPE EXAMPLE 12: A car is uniformly slowed down from 26 $m\ s^{-1}$ to 6 $m\ s^{-1}$ in a period of 8 s. Calculate the acceleration of the car.

$u = 26\ m\ s^{-1}$
$v = 6\ m\ s^{-1}$
$t = 8\ s$
$a = ?$

$$v = u + at$$
$$\therefore a = \frac{v - u}{t} = \frac{6 - 26}{8} = \frac{-20}{8} = -2.5\ m\ s^{-2}$$

i.e. acceleration is $-2.5\ m\ s^{-2}$ (or 2.5 $m\ s^{-2}$ in the opposite direction to the initial motion).

NOTE: When given the rate of slowing down or the retardation assign a negative value to the magnitude (e.g. a body decelerated at 2 $m\ s^{-2}$ has an acceleration of $-2\ m\ s^{-2}$).

SET 13: GRAVITATIONAL ACCELERATION II

NOTE: Use acceleration due to gravity, $g = 10\ m\ s^{-2}$.

1. A stone is thrown vertically downwards at 10 m s^{-1}. After 2.5 s, find the:
   (a) velocity.
   (b) displacement.
2. A ball is thrown vertically downwards into a well with a velocity of 15 m s^{-1}. If the time taken to strike the water is 1 s find the:
   (a) velocity with which the stone strikes the water.
   (b) depth of the well to the water level.
3. A bullet is fired vertically upwards at 600 m s^{-1}.
   What is the velocity of the bullet after:
   (a) 20 s?
   (b) 100 s?
4. A ball is thrown vertically upwards at 13 m s^{-1}. What is its velocity after 1.6 s?
5. A bullet is fired vertically upwards at 600 m s^{-1}. Determine the:
   (a) time taken for the bullet to reach the maximum height
   (b) height reached by the bullet.
6. A stone is thrown vertically upward at 16 m s^{-1}. Calculate the:
   (a) time taken to reach maximum height
   (b) height reached by the stone.
7. A brick falls from rest off the top of a 45 m tall construction site. How long does it take to strike the ground?
8. How long would it take for a parcel to strike the ground if dropped from rest out of a helicopter stationary at a height of 80 m?
9. A stone is dropped from rest. Calculate the:
   (a) time taken to fall 12.8 m
   (b) velocity at this displacement.
10. An object is thrown vertically upward at 17.2 m s^{-1}. How long will it take for the object to return to the ground?

GRAPHICAL REPRESENTATION OF MOTION

A graph will often provide an immediate indication of the type of motion.

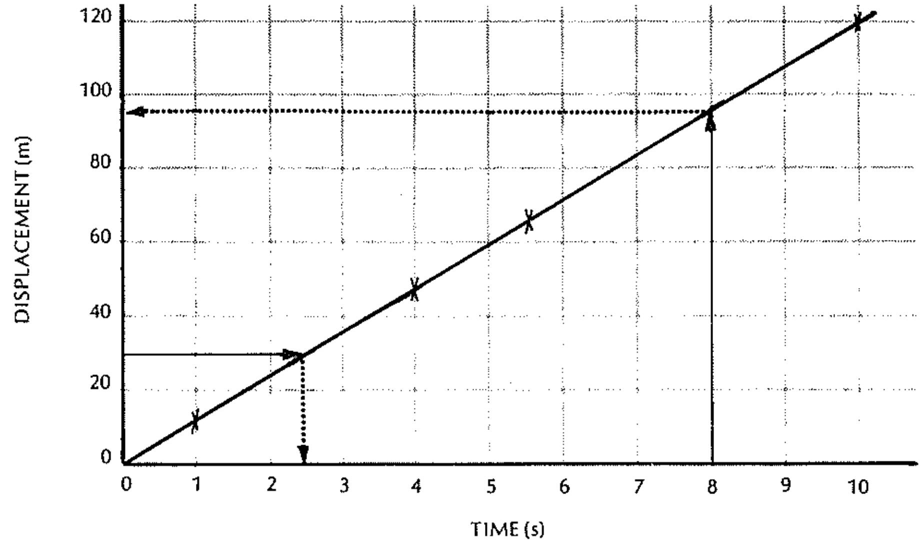
DISPLACEMENT — TIME GRAPHS

TYPE EXAMPLE 17: A motor car travelling along a road provides the following data:

TIME(s)	0	1	4	5.5	10
DISPLACEMENT (m)	0	12	48	66	120

Construct a displacement — time graph and use this graph to estimate the:
(a) displacement after 8 s.
(b) time taken to travel 30 m.

SOLUTION:



NOTE: (1) When constructing a graph from data it is assumed that motion does not vary in between the points plotted.
(2) This graph represents the motion of a car travelling with a uniform velocity of 12 m s^{-1} (travels 120 m in 10 s).
(3) A straight line displacement — time graph always indicates a constant velocity.

(a) To determine displacement after 8 s:
  (i) locate 8 s on the time axis and move (or draw a line) parallel to the displacement axis until the graph representing the motion is intersected.
  (ii) from the point of intersection move (or draw a line) parallel to the time axis till the displacement axis is reached.
  (iii) read off this displacement, in this case 96 m. (See lines drawn on the graph).

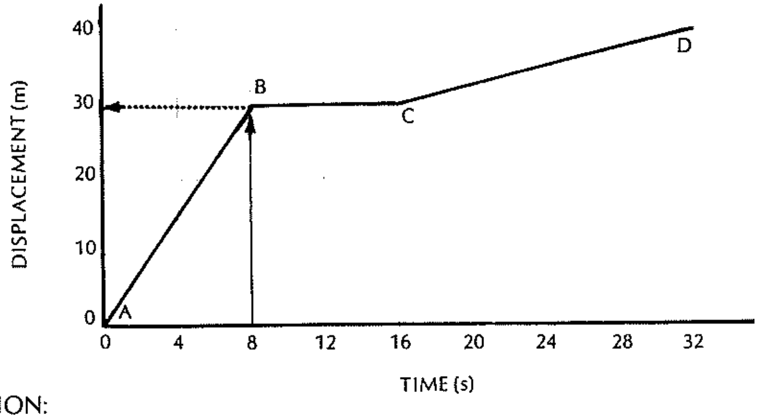
i.e. the displacement after 8 s is 96 m.

(b) To determine the time taken to travel 30 m a similar process to that outlined for (a) gives a time of 2.5 s (see lines drawn on the graph).

i.e. time taken to travel 30 m is 2.5 s.

TYPE EXAMPLE 18: The graph which follows represents a pedestrian walking in a straight line. Use this graph to determine:
(a) how far the pedestrian walked in:
(i) the first 8 s.
(ii) the next 8 s (i.e. between t = 8 s and t = 16 s).

(b) which section of the graph represents the greatest velocity.
(c) the velocity in the first 8 s (i.e. for the portion A B of the graph).
(d) the average velocity for the total 32 s.



SOLUTION:

(a) (i) Distance travelled in the first 8 s is 30 m (see lines on graph).
(ii) Distance travelled in the interval t = 8 s to t = 16 s is 0 m (displacement remains constant at 30 m).

(b) Portion A B represents the greatest velocity (as the graph has the steepest slope in this portion).

(c) To determine velocity in the first 8 s:

$v_{av} = ?$ $\quad s = 30$ m $\quad t = 8$ s

$$v_{av} = \frac{s}{t} = \frac{30}{8} = 3.75 \text{ m s}^{-1}$$

i.e. the velocity for the first 8 s is 3.75 m s^{-1}.

(d) To determine v_{av} for the 32 s:

$v_{av} = ?$ $\quad s = 40$ m $\quad t = 32$ s

$$v_{av} = \frac{40}{32} = 1.25 \text{ m s}^{-1}$$

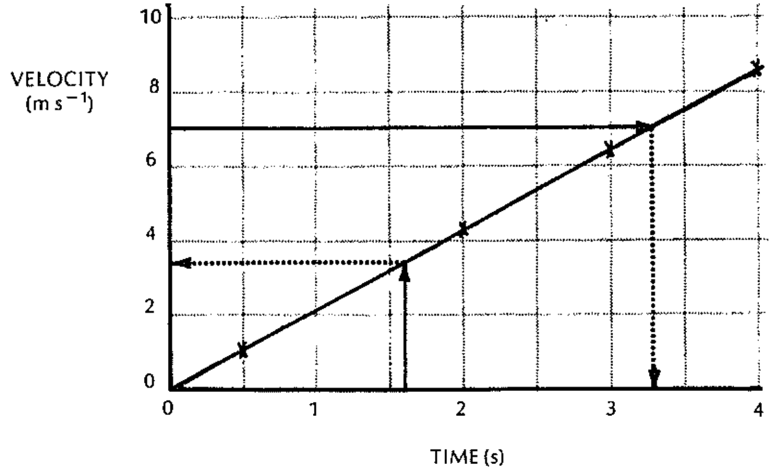
i.e. the average velocity for the 32 s is 1.25 m s^{-1}.

VELOCITY — TIME GRAPHS

TYPE EXAMPLE 19: A cyclist starting from rest has his velocity at various times recorded as follows. Construct a velocity — time graph and determine the:
(a) time taken to reach a velocity of 7 m s^{-1}.
(b) velocity after 1.6 s.

TIME (s)	*0*	*1*	*2*	*3*	*4*
VELOCITY (m s^{-1})	*0*	*2.1*	*4.2*	*6.3*	*8.4*

SOLUTION:

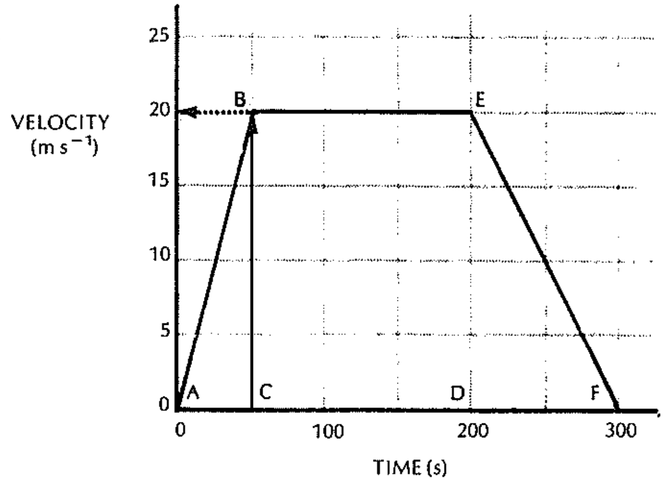


NOTE: 1. A straight line velocity — time graph indicates a constant acceleration.
2. The slope of a velocity — time graph indicates the magnitude of the acceleration (the greater the slope, the greater the acceleration).
3. The area under a velocity — time graph represents the displacement.

From the graph using similar procedures to Type Example 17:

(a) a velocity of 7 m s^{-1} is attained after 3.3 s.
(b) after 1.6 s the cyclist's velocity is 3.4 m s^{-1}.

TYPE EXAMPLE 20: A velocity — time graph is shown below for a train travelling between two stations.



Use this graph to determine the:
(a) velocity of the train after 50 s.
(b) velocity of the train between 50 s and 200 s of the journey.
(c) acceleration of the train between t = 200 s and t = 300 s.
(d) total displacement of the train in 300 s.

SOLUTION:

(a) From the graph, the velocity after 50 s is 20 m s^{-1}.

(b) From the graph, velocity between t = 50 s and t = 200 s remains at 20 m s^{-1} (acceleration is zero as the graph has horizontal slope).

(c) Initial velocity, u (at t = 200s) = 20 m s^{-1}
Final velocity, v (at t = 300 s) = 0 m s^{-1}
Time interval, t = 300 s − 200 s = 100 s
Acceleration, a = ?

$$v = u + at$$
$$\therefore a = \frac{v - u}{t} = \frac{0 - 20}{100} = \frac{-20}{100} = -0.2 \text{ m s}^{-2}$$

i e. the acceleration between t = 200 s and t = 300 s is −0.2 m s^{-2}.

(d) Displacement = area under a v − t graph
= area ABC + area BCDE + area DEF
= ½ (50 × 20) + (150 × 20) + ½ (100 × 20)
= 500 + 3 000 + 1 000
= 4 500 m

i.e. the total displacement is 4 500 m.

SET 14: GRAPHICAL REPRESENTATION OF MOTION

1. Observation of a motor boat provides the following data:

TIME (s)	1	2.5	5	8	10
DISPLACEMENT (m)	4.6	11.5	23	36.8	46

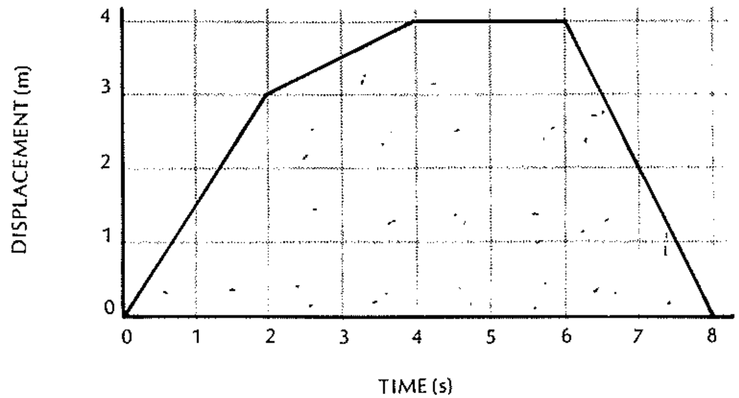
Draw a displacement — time graph to show this motion and use the graph to determine the:
(a) displacement after a
  (i) 9 s period
  (ii) 6.4 s period
(b) time interval to travel
  (i) 15.3 m
  (ii) 35 m

2. Stroboscopic photography of a fast bowler delivering a ball down a cricket pitch gave the following results:

TIME (s)	0.2	0.4	0.6	0.8
DISPLACEMENT (m)	5	10	15	20

Draw a displacement — time graph to determine the:
(a) time taken to deliver the ball the length of the pitch, 20.1 m
(b) distance travelled by the ball in 0.35 s.

3. The graph below represents the motion of a toy electric car.



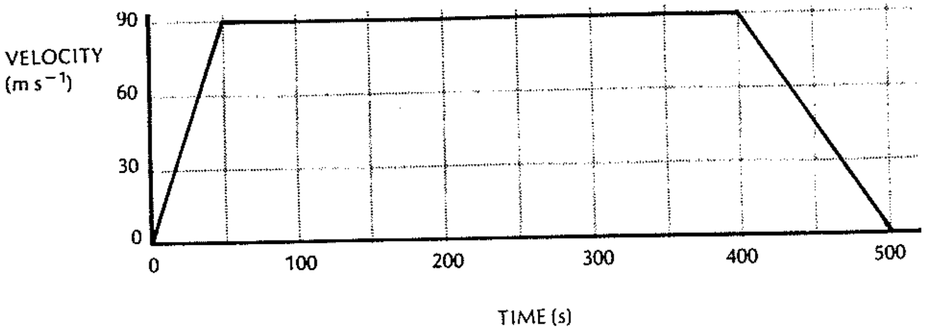
Use this graph to find:
(a) the velocity of the car —
  (i) in the first 2 s
  (ii) in the next 2 s (i.e. between t = 2 s and t = 4 s)
(b) in what time period the car was stationary
(c) the total displacement
(d) the average velocity for the 8 s
(e) the total *distance* travelled
(f) the average *speed* for the 8 s.

4. A stone falling from rest provided the following data:

TIME (s)	0	2	5	7	10
VELOCITY (m s^{-1})	0	19.6	49	68.6	98

Construct a velocity — time graph to estimate the:
(a) velocity of the stone after
  (i) 6.5 s
  (ii) 9 s
(b) time taken to reach a velocity of
  (i) 17 m s^{-1}
  (ii) 80 m s^{-1}

5. A short aeroplane journey is represented by the graph:



Use the graph to find the:

(a) velocity after 50 s
(b) acceleration during the first 50 s
(c) displacement while travelling at a constant velocity
(d) acceleration for the final 100 s of the journey.