PREFACE

"Introduction to Physics CALCULATIONS" is primarily intended for use by students studying Unit 6.4 in Western Australian High Schools. However, it also covers some objectives of Units 6.1 and 3.2.

The book is more than a basic problem book. Sufficient information is included with the numerous type examples and problems to give the student a sound understanding of the topics covered, a realistic view of the quantitative aspects of Physics and a solid foundation upon which further studies in Physics can be developed.

Together with "Introduction to Chemistry CALCULATIONS" by the same authors, the book provides a complete treatment of the quantitative aspects of the science unit curriculum.

CHAPTER 1: Units in Physics

UNITS

Measurements in Physics should use the *INTERNATIONAL SYSTEM OF UNITS (SI).*

This system develops a whole range of units (DERIVED UNITS) from the following BASE UNITS:

QUANTITY	BASE SI UNIT	SYMBOL
length	metre	m
mass	kilogram	kg
time	second	s
temperature	kelvin	K
amount of substance	mole	mol
electric current	ampere	A

Some commonly used DERIVED UNITS are:

QUANTITY	DERIVED SI UNIT	SYMBOL
area	square metres	m^2
volume	cubic metres	m^3
density	kilograms per cubic metre	$kg\ m^{-3}$ (or kg/m^3)
velocity (speed)	metres per second	$m\ s^{-1}$ (or m/s)
acceleration	metres per second squared	$m\ s^{-2}$ (or m/s^2)
force	kilograms metres per second squared	$kg\ m\ s^{-2}$ ($kg\ m/s^2$)

NOTE: The newton (N) is the unit used to measure force. One newton equals one $kg\ m\ s^{-2}$.

Commonly used prefixes:

PREFIX	SYMBOL	MEANING
deci	d	1/10 (10^{-1})
centi	c	1/100 (10^{-2})
milli	m	1/1 000 (10^{-3})
micro	μ	1/1 000 000 (10^{-6})
nano	n	1/1 000 000 000 (10^{-9})
kilo	k	1 000 (10^3)
mega	M	1 000 000 (10^6)
giga	G	1 000 000 000 (10^9)

CONVERSION

Although a range of units are used it is essential that in Physics, base or derived units be used.

TYPE EXAMPLE 1: Determine the number of metres in 2.5 km.

$$2.5\ km = (2.5 \times 1\,000)\ m$$
$$= 2\,500\ m\ (\text{or } 2.5 \times 10^3\ m)$$

TYPE EXAMPLE 2: How many seconds are contained in 1.2 hours?

$$\begin{aligned}1.2 \text{ hours} &= (1.2 \times 60 \times 60)\text{ s}\\ &= 4\,320\text{ s (or } 4.32 \times 10^3\text{ s)}\end{aligned}$$

TYPE EXAMPLE 3: Calculate the number of metres in 7 cm.

$$\begin{aligned}7\text{ cm} &= \frac{7}{100}\text{ m}\\ &= 0.07\text{ m (or } 7 \times 10^{-2}\text{ m)}\end{aligned}$$

SET 1: CONVERSION

Find the number of:

1. metres in 6.2 km
2. seconds in 3.1 minutes
3. seconds in 2.5 hours
4. metres in 176 cm
5. kilograms in 5 500 grams
6. kilograms in 25 grams
7. kilograms in 1.5 tonnes
8. metres in 18 mm
9. square metres (m^2) in 750 square centimetres (cm^2)
10. cubic metres (m^3) in 1 000 cubic centimetres (cm^3)

CHAPTER 2: Motion

MOTION

Motion may be defined as the continuous change in position of an object in relation to another object which is regarded as being at rest.

SCALAR AND VECTOR QUANTITIES

Physical quantities which can be accurately described by the use of a number with the appropriate units are called *SCALAR QUANTITIES (OR SCALARS).*

Scalar quantities have *MAGNITUDE* but *NO DIRECTION.*

Other physical quantities cannot be accurately described in terms of magnitude only. These quantities, which require a direction to be stated, in addition to the magnitude, for an accurate description are called *VECTOR QUANTITIES (OR VECTORS).*

Vector quantities have *MAGNITUDE* and *DIRECTION.*

Examples of scalar and vector quantities:

SCALAR	VECTOR
age (e.g. 25 years)	force (e.g. 5 N south)
time (e.g. 30 s)	displacement (e.g. 52 m south-west)
area (e.g. 8.4 m^2)	velocity (e.g. 4 m s^{-1} east)
distance (e.g. 7.3 m)	acceleration (e.g. 10 m s^{-2} downwards)
mass (e.g. 46 kg)	
speed (e.g. 5 m s^{-1})	
population (e.g. 15 000 000 people)	

REPRESENTATION OF VECTORS

Vector quantities are conveniently represented by arrows. The length of the arrow represents the magnitude of the vector quantity and the direction of the arrow indicates the direction of the vector.

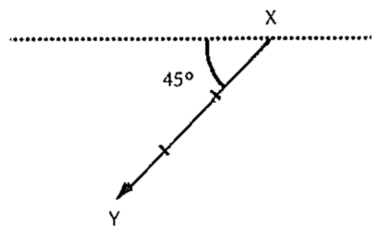
For all graphical representation of vectors clearly indicate the:

1. reference direction (e.g. north).
2. scale (e.g. 1 cm = 10 cm).

TYPE EXAMPLE 4: Show graphically a displacement of 30 m south-west.

Direction: 

Scale: 1 cm = 10 m



XY represents a displacement of 30 m south-west

SET 2: GRAPHICAL REPRESENTATION OF VECTORS

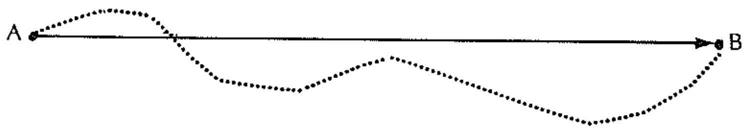
Show graphically each of the following vector quantities:

1. A displacement of 400 m due south.
2. A velocity of 7 m s^{-1} W 30° N.
3. A force of 80 N at 45° to the horizontal.
4. An acceleration of 2 m s^{-2} to the left.
5. A displacement of 24 km north-west.

DISPLACEMENT

Displacement is defined as the straight line distance between the starting and finishing points.

Consider the travel path shown below (.............) between two towns, A and B.



The ....path represents the total *DISTANCE* travelled. The straight line AB, the direct distance between A and B, is the *DISPLACEMENT* in the direction of the line AB.

As displacement is distance in a given direction, it is a vector quantity.

UNIT OF DISTANCE AND DISPLACEMENT.

The unit is that of length, the metre (m).

RESULTANT OF TWO (OR MORE) DISPLACEMENTS

When a body is subjected to two (or more) displacements the final direct distance (the displacement) from the starting position is *NOT* equal to the total distance travelled (unless the individual displacements are in the same direction).

To determine the exact location (distance and direction from the starting position) of a body, it is necessary to determine the *RESULTANT* displacement.

A resultant of two (or more) vectors may be defined as the single vector which on its own will produce the same effect as the two (or more) vectors combined.

DETERMINATION OF RESULTANTS

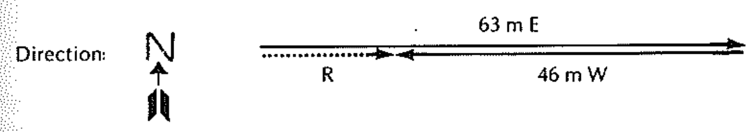
To determine the resultant:

1. Place the vectors head to tail (in any order).
2. Draw in the resultant which is from the tail of the first vector to the head of the last.
3. If the vectors have been drawn to scale then the magnitude of the resultant and the direction can be measured — otherwise both can be calculated.

This method is often referred to as the *"head to tail"* or *"triangle"* method.

TYPE EXAMPLE 5: What is the resultant displacement of a displacement of 63 m east and a displacement of 46 m west?

Sketch vectors head to tail (approximately to scale) and the resultant displacement.



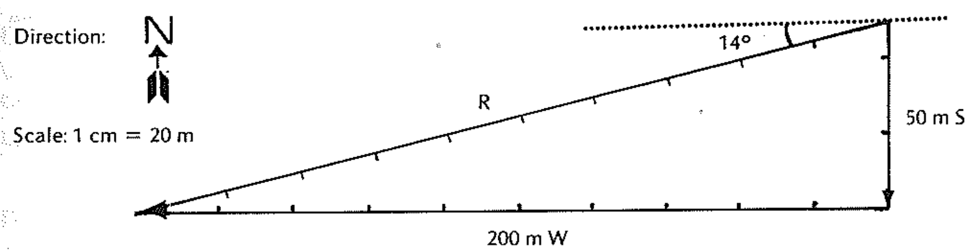
For displacements in one straight line, measurement (i.e. graphical solution) is unnecessary as the calculation of the magnitude of the resultant is obvious.

$$\text{resultant, } R = (63 \text{ m}) - (46 \text{ m}) = 17 \text{ m}$$

i.e. the resultant displacement is 17 m east.

TYPE EXAMPLE 6: Determine the resultant displacement if a person walks 50 m south to a corner and then 200 m west to go to the local store.

A. GRAPHICAL SOLUTION

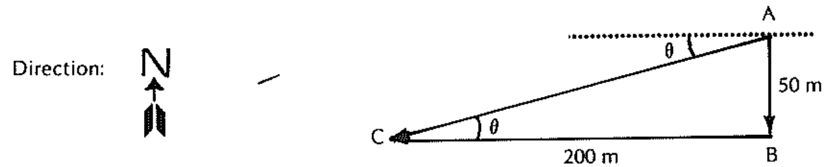


$$\text{length of } R = 10.3 \text{ cm}$$
$$\therefore \text{magnitude of } R = (10.3 \times 20) \text{ m} = 206 \text{ m}$$

i.e. the resultant displacement is 206 m W 14° S

B. CALCULATION SOLUTION (right angled triangles only).

1. Sketch the vector triangle (approximately to scale).



2. Using Pythagorus' Theorem calculate a value for the magnitude of the resultant, AC.

AC = resultant
= ?
AB = 50 m S
BC = 200 m W

$$AC^2 = AB^2 + BC^2$$
$$= (50)^2 + (200)^2$$
$$= 2\,500 + 40\,000$$
$$= 42\,500$$
$$\therefore AC = 206.2 \text{ m}$$

3. Using trigonometric ratios determine the direction of the resultant.

$$\tan\theta = \frac{AB}{BC}$$
$$= \frac{50}{200}$$
$$= 0.25$$
$$\therefore \theta = 14°$$

i.e. the resultant displacement is 206.2 m W 14° S.

SET 3: RESULTANT DISPLACEMENT I

Determine the resultant displacement of:

1. A run of 52 m due south followed by a run of 164 m due south.
2. A football kicked 35 m directly down field then punched a further 12 m in the same direction.
3. 65 m north, 78 m south.
4. A drive of 164 km SW, then 49 km NE.
5. A ball dropped from a building 25 m high onto the ground, bouncing vertically upwards and caught 2 m above the ground.

SET 4: RESULTANT DISPLACEMENT II

Determine graphically the resultant displacement for each of the following:

1. A boy walks 200 m due south, then 100 m south-west.
2. A plane flies 250 km W then 500 km S.
3. 40 cm to the left, 20 cm vertically upwards.
4. A ball rolling 50 cm down an incline (30° to the horizontal) and then 30 cm along the horizontal.
5. 14 m N, 10 m W, 5 m SW.

SET 5: RESULTANT DISPLACEMENT III

Using Pythagorus' Theorem and trigonometric ratios determine the resultant displacement for each of the following:

1. 300 km N, 300 km W.
2. A climb 6 m out of a well and 8 m away from it.
3. 500 m E, 1 200 m S.
4. 17.32 m horizontally then 10 m vertically upwards.
5. 1.2 km NW, 1.6 km NE.

SPEED

The speed of a body is the rate of change of distance by the body.

The average speed of a body over a distance travelled in a time interval can be determined by:

$$\boxed{\text{Speed (av)} = \frac{d}{t}}$$

where d = distance (measured from zero)
t = time interval (measured from zero)

Speed is a scalar quantity as it involves no specific direction. Distance is the total length of the path covered by the body.

If to accurately describe motion direction is required, then *VELOCITY* is used in place of speed.

VELOCITY

The velocity of a body is the rate of change of displacement of the body.

The average velocity of a body over a displacement in a measured time interval can be determined by:

$$\boxed{v_{av} = \frac{s}{t}}$$

v_{av} = average velocity
s = displacement (measured from zero)
t = time interval (measured from zero)

Velocity is a vector quantity. *The direction of the velocity is that of the displacement (unless otherwise stated).*

UNITS OF VELOCITY AND SPEED

Velocity and speed are both determined by dividing units of length (m) by units of time (s).

Units: $\frac{\text{metres}}{\text{seconds}} = \frac{m}{s} = m\ s^{-1}$ (or m/s)

DETERMINATION OF DISPLACEMENT

The equation for average velocity can be rearranged to determine the displacement.

$$v_{av} = \frac{s}{t}$$

rearranging gives: $s = v_{av} \times t$
similarly $d = \text{speed (av)} \times t$

TYPE EXAMPLE 7: Calculate the average velocity of a vehicle which travels 3 km east in 5 minutes.

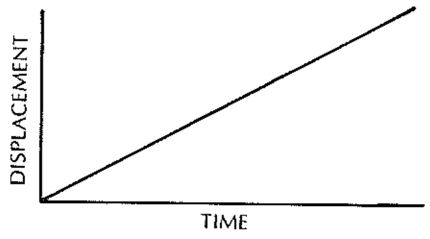
$v_{av} = ?$
$s = 3$ km
$= (3 \times 1\,000)$ m
$t = 5$ minutes
$= (5 \times 60)$ s

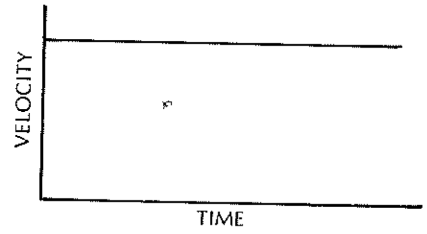
$$v_{av} = \frac{s}{t}$$
$$= \frac{3 \times 1\,000}{5 \times 60}$$
$$= 10\ m\ s^{-1}$$

i.e. the average velocity is 10 m s^{-1} east.

NOTE: 1. While speed and direction remain unchanged the velocity remains the same.

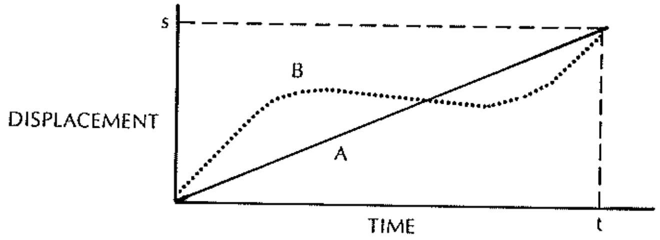
2. The graphs below represent motion with a velocity which does not vary during the considered period of time.





Velocity which does not vary is termed CONSTANT or UNIFORM VELOCITY.

3. Consider the two motions represented below.



Motions A and B both represent a displacement s in a time interval t.

$\therefore$ in each case v_{av} is the same (s/t)

Motion A is uniform (or constant) velocity while motion B is a varying velocity. Therefore for motion A, the velocity at any instant is equal to the average velocity which is equal to the uniform (or constant) velocity.

TYPE EXAMPLE 8: Determine the displacement of a body which moves with a uniform velocity of 2.4 m s⁻¹ for 50 s.

$v_{av} = 2.4\ m\ s^{-1}$
$s = ?$
$t = 50\ s$

$$v_{av} = \frac{s}{t}$$
$$\therefore \quad s = v_{av} \times t = 2.4 \times 50 = 120\ m$$

i.e. the displacement is 120 m in the direction of the velocity.

SET 6: VELOCITY I

1. What is the average speed of a sprinter who runs 100 m in 12.5 s?
2. A car takes 80 s to complete one lap of a 2 km circuit. What is its average speed (in $m\ s^{-1}$)?
3. A body travels 36 m in 4 s. What is its average speed?
4. How far will a bird fly in 25 s travelling with an average speed of 6 $m\ s^{-1}$?
5. A person swims from one end of a pool to the other at an average velocity of 1.5 $m\ s^{-1}$ in 20 s. What is the length of the pool?
6. Determine the distance travelled by a plane in 1 hour if it flies due south with a uniform velocity of 90 $m\ s^{-1}$.
7. How long will it take for a body moving with a uniform velocity of 7 $m\ s^{-1}$ to cover 98 m?
8. A cricket ball is thrown a distance of 90 m from the boundary to the wicket at an average horizontal speed of 30 $m\ s^{-1}$. How long will it take for the ball to reach the wicket?
9. Determine the time taken for a horse running with an average speed of 14 $m\ s^{-1}$ to complete one circuit of a 840 m track.
10. How long will it take to complete a trip from Perth to Kalgoorlie, a distance of 595 km, travelling at an average speed of 70 $km\ h^{-1}$?
11. A train moving with an average speed of 80 $km\ h^{-1}$ takes 0.25 hours to travel from one railway siding to the next. What is the distance between the two sidings?
12. Determine the average speed ($km\ h^{-1}$) of a plane which flies a distance of 720 km in 1.5 hours?

SET 7: VELOCITY II

1. An athlete, commencing at a point due east, completes half a lap of a 440 m track in 25 s. Determine the athlete's:
   (a) average speed.
   (b) average velocity (in westerly direction).
   (c) average velocity if the athlete completes one full lap at the same pace.
2. A car travels with an average speed of 20 $m\ s^{-1}$ due north for 3 km then due west for another 4 km calculate:
   (a) the total time taken for the journey.
   (b) the displacement of the car.
   (c) the average velocity of the car.
3. A person walks for 10 s at 1 $m\ s^{-1}$ then for another 10 s in the same direction at 2 $m\ s^{-1}$. Find the:
   (a) total displacement.
   (b) average velocity.
4. A body travels 200 m at a uniform velocity of 4 $m\ s^{-1}$ and a further 150 m in the same direction at a uniform velocity of 7.5 $m\ s^{-1}$. Determine the:
   (a) total time taken.
   (b) average velocity for the whole journey.
5. A cyclist travels at 6 $m\ s^{-1}$ S for 10 minutes and then returns 600 m along the same path. What is the final displacement of the cyclist?

6. In a 50 m swimming race the winner completed the race in 32 seconds. By what margin (length) did the winner defeat the second placegetter who swam with an average velocity of 1.5 m s^{-1}?
7. A cyclist moving uniformly at 5 m s^{-1} overtakes a pedestrian walking in the same direction in 6 s. If the pedestrian initially was 18 m ahead of the cyclist, find the pedestrian's average velocity.
8. Determine the time saved by travelling 8 km at 40 m s^{-1} rather than at 32 m s^{-1}.
9. How far will a car moving at 20 m s^{-1} travel in the same time that a car moving at 25 m s^{-1} travels 1 km?
10. A car travelling at 15 m s^{-1} is 10 m behind another vehicle moving in the same direction. If it takes 10 s to overtake this vehicle, what is the speed of the slower vehicle?

ACCELERATION

Acceleration occurs whenever the velocity of a body changes.

Acceleration is defined as the rate of change of velocity.

For the purpose of examples considered in this text acceleration is considered to be uniform (or constant).

From the definition:

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$= \frac{\text{final velocity} - \text{initial velocity}}{\text{time}}$$

$$a(av) = \frac{v - u}{t}$$

rearranging gives: $\boxed{v = u + at}$ (assuming uniform acceleration)

where: v = final velocity
u = initial velocity
a = uniform acceleration
t = time interval (measured from zero)

UNITS OF ACCELERATION

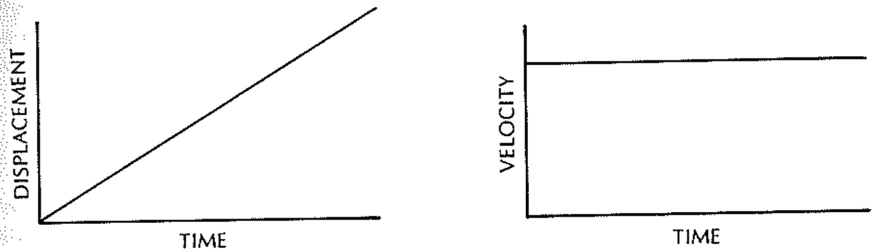
As acceleration is the rate of change of velocity then its units are those of velocity divided by time.

$$\text{Units:} \quad \frac{m\ s^{-1}}{s} = m\ s^{-2}\ (\text{or } m/s^2)$$

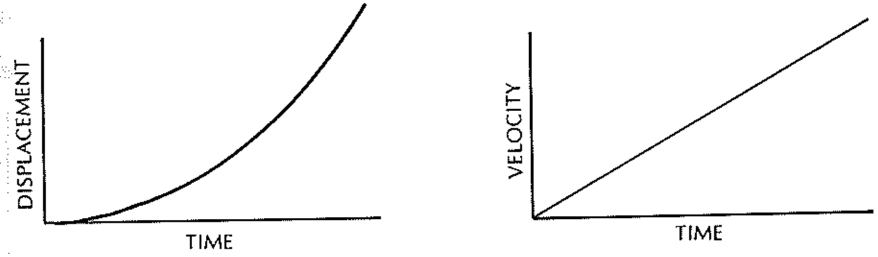
GRAPHICAL REPRESENTATION OF ACCELERATED MOTION

Consider the following graphical representations of motion:

(a) Uniform velocity (zero acceleration)



(b) Uniform acceleration



STRAIGHT LINE MOTION

If the motion being considered is in one straight line, then vectors (i.e. displacement, acceleration and velocity) can only have one of two directions.

The following sign convention for vector directions will be used:

1. *Consider the direction of initial motion to be positive. All vectors in this direction will be given a positive value.*
2. *Assume any stated acceleration to be in the direction of the initial motion and hence positive, unless stated otherwise.*
3. *If an acceleration is found to be*
   (a) positive, then it is in the direction of the initial motion and velocity increases uniformly.
   (b) negative, then it is in the opposite direction to the initial motion and velocity decreases uniformly.

NOTE: For a body starting from rest, the initial velocity, u = 0.

TYPE EXAMPLE 9: A car starting from rest is accelerated at 2 m s^{-2}. Calculate the velocity after 5 s.

$u = 0$
$v = ?$
$a = 2\ m\ s^{-2}$
$t = 5\ s$

$v = u + at$
$= 0 + (2 \times 5)$
$= 10\ m\ s^{-1}$

i.e. the velocity is 10 m s^{-1} (in the direction of the acceleration).

TYPE EXAMPLE 10: Determine the acceleration of a body which is accelerated from 2 m s⁻¹ to 10 m s⁻¹ in 2.5 s.

$u = 2\ m\ s^{-1}$
$v = 10\ m\ s^{-1}$
$a = ?$
$t = 2.5\ s$

$$v = u + at$$
$$\therefore a = \frac{v - u}{t} = \frac{(10 - 2)}{2.5} = \frac{8}{2.5} = 3.2\ m\ s^{-2}$$

i.e. acceleration is $3.2\ m\ s^{-2}$ (in the direction of the initial motion).

TYPE EXAMPLE 11: What is the initial velocity of a body which attained a velocity of 16 m s⁻¹ when accelerated at 1.5 m s⁻² for 8 s?

$u = ?$
$v = 16\ m\ s^{-1}$
$a = 1.5\ m\ s^{-2}$
$t = 8\ s$

$$v = u + at$$
$$\therefore u = v - at = 16 - (1.5 \times 8) = 16 - 12 = 4\ m\ s^{-1}$$

i.e. the initial velocity is $4\ m\ s^{-1}$ (in the direction of the final velocity)

SET 8: ACCELERATION

1. What is the increase in velocity in each second of a body accelerated at $1.8\ m\ s^{-2}$?
2. Calculate the final velocity of a motor car initially travelling at $10\ m\ s^{-1}$ which is accelerated at $1.4\ m\ s^{-2}$ for 10 s.
3. A body starts from rest and is accelerated at $2.7\ m\ s^{-2}$. What is the velocity after 12 s?
4. A ball rolling down a slope from rest is accelerated at $3.5\ m\ s^{-2}$. What is the ball's velocity after 6 s?
5. Find the acceleration of a plane which increases its velocity from $100\ m\ s^{-1}$ to $200\ m\ s^{-1}$ in 40 s.
6. What is the average acceleration of a body which is accelerated from rest to $15\ m\ s^{-1}$ in 5 s?
7. Determine the average acceleration experienced by a body which increases its velocity from $7\ m\ s^{-1}$ to $34\ m\ s^{-1}$ in 9 s.
8. A vehicle starting from rest is accelerated at $2.5\ m\ s^{-2}$. Determine the time taken to reach a velocity of $5\ m\ s^{-1}$.
9. Find the time taken for a body travelling with a velocity of $15\ m\ s^{-1}$ to reach a velocity of $29\ m\ s^{-1}$ when accelerated at $0.7\ m\ s^{-2}$.
10. A car is accelerated from $10\ m\ s^{-1}$ to $25\ m\ s^{-1}$ in 2.5 s. What is the average acceleration?

NEGATIVE ACCELERATION

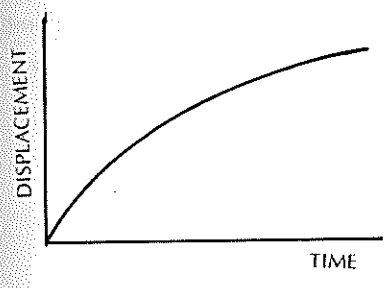
If a body (travelling in the positive direction) slows down uniformly it can be seen from the equation derived from the definition of acceleration that the value obtained for acceleration will be negative.

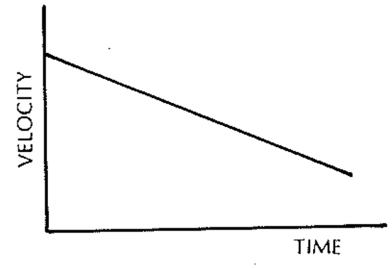
That is, as $a = \frac{v - u}{t}$ then if $v < u$, a will be negative.

Whenever the velocity vector and the acceleration vector are in opposite directions the magnitude of the velocity decreases uniformly.

A negative acceleration is also referred to as *RETARDATION* or *DECELERATION*.

GRAPHICAL REPRESENTATION OF UNIFORM NEGATIVE ACCELERATION





TYPE EXAMPLE 12: A car is uniformly slowed down from 26 m s⁻¹ to 6 m s⁻¹ in a period of 8 s. Calculate the acceleration of the car.

$u = 26\ m\ s^{-1}$
$v = 6\ m\ s^{-1}$
$t = 8\ s$
$a = ?$

$$v = u + at$$
$$\therefore a = \frac{v - u}{t} = \frac{6 - 26}{8} = \frac{-20}{8} = -2.5\ m\ s^{-2}$$

i.e. acceleration is $-2.5\ m\ s^{-2}$ (or $2.5\ m\ s^{-2}$ in the opposite direction to the initial motion).

NOTE: When given the rate of slowing down or the retardation assign a negative value to the magnitude (e.g. a body decelerated at $2\ m\ s^{-2}$ has an acceleration of $-2\ m\ s^{-2}$).

SET 9: NEGATIVE ACCELERATION

1. A train approaching a railway station at 30 m s^{-1} is brought to rest in 10 s. Determine the retardation.
2. A plane on landing touches down at a speed of 80 m s^{-1}. Determine the acceleration if the plane comes to rest in a time of 8 s.
3. A truck travelling at 36 m s^{-1} is decelerated uniformly at 1.5 m s^{-2}. Calculate the: (a) velocity after 6 s.
   (b) time taken to come to rest.
4. A plane flying at 170 m s^{-1} slows down uniformly at 2 m s^{-2}. What is the plane's velocity after 30 s?
5. How long will it take a vehicle travelling at 30 m s^{-1} to reach a velocity of 12 m s^{-1} if the acceleration is -3 m s^{-2}?
6. A bus travelling at 25 m s^{-1} is decelerated at 2.5 m s^{-2}. Find the time taken for the bus to come to rest.
7. A motor cyclist uniformly decelerates at 3 m s^{-2}. If the initial velocity is 32 m s^{-1}, calculate the time taken to attain a velocity of 14 m s^{-1}.
8. A cyclist brakes suddenly with a uniform acceleration of -4 m s^{-2} and comes to rest in 2.5 s. Determine the initial velocity of the cyclist.

DISPLACEMENT OF UNIFORMLY ACCELERATED BODIES

For any moving body: $v_{av} = \frac{s}{t}$

For a body moving with a uniform acceleration:

$$v_{av} = \frac{u+v}{2}$$

hence: $$\frac{s}{t} = \frac{u+v}{2}$$
$$= \frac{u+(u+at)}{2} \quad \text{(as } v = u + at\text{)}$$

$$\therefore \boxed{s = ut + ½at^2}$$

TYPE EXAMPLE 13: A vehicle travelling at 10 m s^{-1} is accelerated at 4 m s^{-2}. After a period of 5 s, determine the:
(a) displacement
(b) average velocity
(c) final velocity.

(a) $u = 10$ m s^{-1}, $a = 4$ m s^{-2}, $t = 5$ s, $s = ?$

$$\begin{aligned} s &= ut + ½at^2 \\ &= (10 \times 5) + ½(4 \times 5^2) \\ &= 50 + ½(4 \times 25) \\ &= 50 + 50 \\ &= 100 \text{ m} \end{aligned}$$

i.e. the displacement is 100 m (in the direction of the initial motion).

(b) $v_{av} = ?$, $s = 100$ m, $t = 5$ s

$$\begin{aligned} v_{av} &= \frac{s}{t} \\ &= \frac{100}{5} \\ &= 20 \text{ m s}^{-1} \end{aligned}$$

i.e. the average velocity is 20 m s^{-1} (in the direction of the initial motion).

(c) $v = ?$, $u = 10$ m s^{-1}, $a = 4$ m s^{-2}, $t = 5$ s

$$\begin{aligned} v &= u + at \\ &= 10 + (4 \times 5) \\ &= 10 + 20 \\ &= 30 \text{ m s}^{-1} \end{aligned}$$

i.e. the final velocity is 30 m s^{-1} (in the direction of the initial motion).

OR $v = ?$, $u = 10$ m s^{-1}, $v_{av} = 20$ m s^{-1}

$$\begin{aligned} v_{av} &= \frac{u+v}{2} \\ \therefore u + v &= 2 \times v_{av} \\ \therefore v &= (2 \times v_{av}) - u \\ &= (2 \times 20) - 10 \\ &= 40 - 10 \\ &= 30 \text{ m s}^{-1} \end{aligned}$$

i.e. the final velocity is 30 m s^{-1} (in the direction of the initial motion).

SET 10: DISPLACEMENT OF UNIFORMLY ACCELERATED BODIES I

1. A car moving with a velocity of 20 m s^{-1} is accelerated at 3 m s^{-2}. After 8 s, determine the:
   (a) displacement.
   (b) average velocity.
   (c) final velocity.
2. What distance will a cyclist travel if when starting from rest the cycle is accelerated at 1.8 m s^{-2} for 4 s?
3. A motor boat moving with a velocity of 2 m s^{-1} is accelerated at 1 m s^{-2} for 6 s. Calculate the:
   (a) displacement.
   (b) final velocity.
   (c) average velocity.
4. A train, initially at rest, leaves a railway station with an acceleration of 0.4 m s^{-2} for a period of 50 s. Find the:
   (a) displacement.
   (b) final velocity.
   (c) average velocity.
5. A truck travelling at 15 m s^{-1} brakes uniformly at 1.75 m s^{-2} for 4 s in coming to rest at a stop sign. At what distance from the sign did the truck commence braking?

6. A ball starting from rest rolls down an inclined plank with an average acceleration of 5 m s^{-2}. If it takes 2 s to roll off the end, determine the:
   (a) length of the plank.
   (b) final velocity of the ball.
   (c) average velocity of the ball.
7. A bicycle moving at 8 m s^{-1} is accelerated at -1.25 m s^{-2} for a 6 s period. Determine the:
   (a) displacement.
   (b) velocity at the end of this period.
8. A body with a speed of 50 m s^{-1} is subjected to a uniform retardation of 10 m s^{-2} for 4 s. Determine the:
   (a) distance travelled.
   (b) average speed.
   (c) final speed.
9. A plane can effect take-off with a velocity of 72 m s^{-1}. If the plane can accelerate along the runway at 4 m s^{-2}, determine the:
   (a) time taken to reach a speed of 72 m s^{-1}
   (b) minimum length of runway to enable a safe take-off.
10. A person starting from rest takes 6 s to reach a maximum velocity down a ski slope. Assuming a constant acceleration of 5 m s^{-2} find the:
   (a) distance travelled in 6 s
   (b) maximum velocity attained by the skier.

SET 11: DISPLACEMENT OF UNIFORMLY ACCELERATED BODIES II

1. A rocket is uniformly accelerated from rest at 8 m s^{-2} for a period of 12 s. Find the:
   (a) final velocity
   (b) displacement
   (c) average velocity.
2. A car is accelerated from 17 m s^{-1} to 44 m s^{-1} in 18 s. Calculate the:
   (a) average velocity
   (b) acceleration
   (c) displacement in this period.
3. Determine the time taken for a vehicle accelerated from rest at 2 m s^{-2} to travel a distance of 36 m.
4. What period of time is required for a plane starting from rest and accelerated at 2.8 m s^{-2} to travel a distance of 875 m to effect take-off?
5. A car travelling at 42 m s^{-1} can be stopped in 7 s. Find the:
   (a) acceleration
   (b) minimum distance required to stop the car.
6. An athlete starting from rest attains his maximum speed in a distance of 9.6 m in a period of 2 s. Calculate the:
   (a) acceleration (assumed to be constant)
   (b) average speed
   (c) maximum speed.
7. A boat commencing from rest is accelerated uniformly at 1.5 m s^{-2}. Determine the displacement in the 5th second of travel.
8. What distance does a cyclist travel in the 4th second of motion if when moving at 2 m s^{-1} he accelerates at 0.8 m s^{-2}?

9. Determine the distance required for a car to accelerate from rest to 72 km h^{-1} 20 s.
10. How far will a train travel if slowed from 46 m s^{-1} to 22 m s^{-1} in 8 s?

ANOTHER EQUATION OF MOTION

The equation, $v^2 = u^2 + 2as$ is often used to solve in one step problems which quire the use of both of the other two equations of motion, $v = u + at$, a $s = ut + ½ at^2$.

GRAVITATIONAL ACCELERATION

All freely falling bodies close to the earth's surface are accelerated at the same ra (i.e. they fall the same distance in the same time). The acceleration due to gravity often referred to as g.

Acceleration due to gravity acts in a vertically downward direction and has a val of approximately 10 m s^{-2}.

The previously established equations of motion for uniformly accelerated bod may be used for freely falling bodies.

$$v = u + gt$$
$$s = ut + ½ gt^2$$

where: g = acceleration due to gravity
= 10 m s^{-2} (vertically downward)

NOTE: 1. The effects of frictional air resistance are ignored and are assumed to negligible.
2. At ground level g has a value of about 9.8 m s^{-2}. For ease of calculation t rounded off value of 10 m s^{-2} is used.

TYPE EXAMPLE 14: A stone falls off the edge of a cliff. If it takes 6 s to strike ground below, determine the:
(a) velocity with which the stone strikes the ground.
(b) height of the cliff.

(a) $u = 0$, $t = 6$ s, $g = 10$ m s^{-2}, $v = ?$

$$v = u + gt = 0 + (10 \times 6) = 60 \text{ m s}^{-1}$$

i.e. the stone strikes the ground at 60 m s^{-1} (downwards).

(b) $u = 0$, $t = 6$ s, $g = 10$ m s^{-2}, $s = ?$

$$s = ut + ½ gt^2 = 0 + ½ (10 \times 6^2) = ½ (10 \times 36) = 180 \text{ m}$$

i.e. as the displacement is 180 m (downwards) the height of the cliff is 180 m.

SET 12: GRAVITATIONAL ACCELERATION I

NOTE: Use acceleration due to gravity, $g = 10\ m\ s^{-2}$.

1. A stone drops freely from rest. Determine after 4 s the:
   (a) stone's final velocity
   (b) displacement.
2. A can of paint falls from the top of a painter's ladder.
   If it takes 1.2 s for the can to strike the ground below, find the:
   (a) final velocity of the can
   (b) length of the ladder (assumed to be vertical).
3. A cricket ball is hit vertically into the air. The ball at its maximum height is momentarily at rest and from this position takes 2.5 s to strike the ground. Calculate the height to which the ball was hit.
4. A stone is dropped from rest into a well. Find the depth of the well if it takes 0.9 s for the stone to strike the bottom.
5. A ball is thrown vertically downwards at 10 m s^{-1}. How far does the ball travel in 3 s?
6. An object drops (from rest) out of a window of a tall building. How far does the object fall:
   (a) in 2 s?
   (b) in 3 s?
   (c) in the 3rd second?

ACCELERATION DUE TO GRAVITY (EXTENDED)

When a projectile has an initial velocity upwards the sign convention *MUST* be used as 'g' is *ALWAYS* directed vertically downards. As it rises the projectile slows until at its maximum height its velocity is *MOMENTARILY* zero. This occurs because the initial velocity and acceleration vectors are in opposite directions. From its maximum height the projectile falls with a velocity increasing in the downward direction until, at the moment it returns to the position from which it was projected, it is of the same magnitude as when it was projected vertically but in the *OPPOSITE* direction. This occurs because the final velocity and acceleration vectors are in the same direction.

For the entire journey g is constant (and equal to 10 m s^{-2} downwards).

Generally downward vectors are designated a *POSITIVE* sign.

TYPE EXAMPLE 15: A stone is thrown vertically upward with a velocity of 35 m s^{-1}. Find the velocity of the stone after

(a) 2 s

(b) 4 s.

(a) $g = +10\ m\ s^{-2}$, $u = -35\ m\ s^{-1}$, $t = 2\ s$, $v = ?$

$$\begin{aligned} v &= u + gt \\ &= -35 + (10 \times 2) \\ &= -35 + 20 \\ &= -15\ m\ s^{-1} \end{aligned}$$

i.e. the velocity after 2 s is 15 m s^{-1} UPWARDS.

(b) $g = +10\ m\ s^{-2}$, $u = -35\ m\ s^{-1}$, $t = 4\ s$, $v = ?$

$$\begin{aligned} v &= u + gt \\ &= -35 + (10 \times 4) \\ &= -35 + 40 \\ &= 5\ m\ s^{-1} \end{aligned}$$

i.e. the velocity after 4 s is 5 m s^{-1} DOWNWARDS.

To determine the maximum height reached by a projectile during its flight the following may assist:

1. *at maximum height, the velocity is momentarily zero.*
2. *the time taken to reach maximum height is the same as the time taken to return from the position of maximum height.*

TYPE EXAMPLE 16: An arrow is fired vertically upwards with a velocity of 80 m s^{-1}.

Determine the: (a) time taken to reach maximum height.
(b) maximum height reached by the arrow.

(a) $g = +10\ m\ s^{-2}$, $u = -80\ m\ s^{-1}$, $v = 0$, $t = ?$

$$\begin{aligned} v &= u + gt \\ \therefore gt &= v - u \\ \therefore t &= \frac{v-u}{g} \\ &= \frac{0-(-80)}{10} \\ &= 8\ s \end{aligned}$$

i.e. the time taken to reach maximum height is 8 s.

(b) *Consider the arrow falling from its position of maximum height.*

The time taken to reach maximum height is equal to the time taken for the arrow to return $\therefore t = 8$ s

$g = 10\ m\ s^{-2}$, $t = 8\ s$, $u = 0$, $s = ?$

$$\begin{aligned} s &= ut + \tfrac{1}{2} gt^2 \\ &= 0 + \tfrac{1}{2}(10 \times 8^2) \\ &= \tfrac{1}{2}(10 \times 64) \\ &= 320\ m \end{aligned}$$

i.e. As the displacement of the falling arrow is 320 m (DOWNWARDS) the maximum height reached by the arrow is 320 m.

OR

Consider the arrow rising to its position of maximum height.

$s = ?$, $g = 10\ m\ s^{-2}$, $u = -80\ m\ s^{-1}$, $t = 8$ s (from part a)

$$\begin{aligned} s &= ut + \tfrac{1}{2} gt^2 \\ &= (-80) \times 8 + \tfrac{1}{2}(10 \times 8^2) \\ &= -640 + 320 \\ &= -320\ m \end{aligned}$$

i.e. As the displacement is 320 m (UPWARDS) the maximum height reached by the arrow is 320 m.

SET 13: GRAVITATIONAL ACCELERATION II

NOTE: Use acceleration due to gravity, $g = 10\ m\ s^{-2}$.

1. A stone is thrown vertically downwards at 10 m s^{-1}. After 2.5 s, find the:
   (a) velocity.
   (b) displacement.
2. A ball is thrown vertically downwards into a well with a velocity of 15 m s^{-1}. If the time taken to strike the water is 1 s find the:
   (a) velocity with which the stone strikes the water.
   (b) depth of the well to the water level.
3. A bullet is fired vertically upwards at 600 m s^{-1}.
   What is the velocity of the bullet after:
   (a) 20 s?
   (b) 100 s?
4. A ball is thrown vertically upwards at 13 m s^{-1}. What is its velocity after 1.6 s?
5. A bullet is fired vertically upwards at 600 m s^{-1}. Determine the:
   (a) time taken for the bullet to reach the maximum height
   (b) height reached by the bullet.
6. A stone is thrown vertically upward at 16 m s^{-1}. Calculate the:
   (a) time taken to reach maximum height
   (b) height reached by the stone.
7. A brick falls from rest off the top of a 45 m tall construction site. How long does it take to strike the ground?
8. How long would it take for a parcel to strike the ground if dropped from rest out of a helicopter stationary at a height of 80 m?
9. A stone is dropped from rest. Calculate the:
   (a) time taken to fall 12.8 m
   (b) velocity at this displacement.
10. An object is thrown vertically upward at 17.2 m s^{-1}. How long will it take for the object to return to the ground?

GRAPHICAL REPRESENTATION OF MOTION

A graph will often provide an immediate indication of the type of motion.

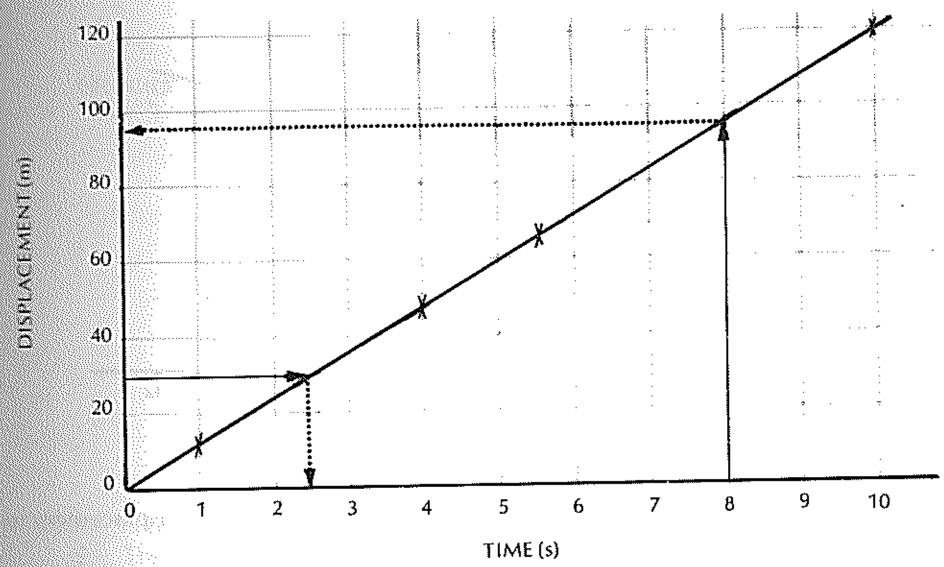
DISPLACEMENT — TIME GRAPHS

TYPE EXAMPLE 17: A motor car travelling along a road provides the following data:

TIME(s)	0	1	4	5.5	10
DISPLACEMENT (m)	0	12	48	66	120

Construct a displacement — time graph and use this graph to estimate the:
(a) displacement after 8 s.
(b) time taken to travel 30 m.

SOLUTION:



NOTE: (1) When constructing a graph from data it is assumed that motion does not vary in between the points plotted.
(2) This graph represents the motion of a car travelling with a uniform velocity of 12 m s^{-1} (travels 120 m in 10 s).
(3) A straight line displacement — time graph always indicates a constant velocity.

(a) To determine displacement after 8 s:
   (i) locate 8 s on the time axis and move (or draw a line) parallel to the displacement axis until the graph representing the motion is intersected.
   (ii) from the point of intersection move (or draw a line) parallel to the time axis till the displacement axis is reached.
   (iii) read off this displacement, in this case 96 m. (See lines drawn on the graph).

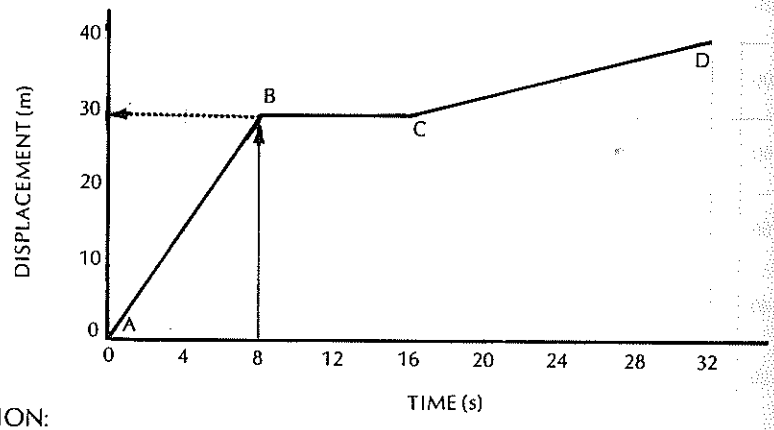
i.e. the displacement after 8 s is 96 m.

(b) To determine the time taken to travel 30 m a similar process to that outlined for (a) gives a time of 2.5 s (see lines drawn on the graph).

i.e. time taken to travel 30 m is 2.5 s.

TYPE EXAMPLE 18: The graph which follows represents a pedestrian walking in a straight line. Use this graph to determine:
(a) how far the pedestrian walked in:
   (i) the first 8 s.
   (ii) the next 8 s (i.e. between $t = 8$ s and $t = 16$ s).

(b) *which section of the graph represents the greatest velocity.*
(c) *the velocity in the first 8 s (i.e. for the portion A B of the graph).*
(d) *the average velocity for the total 32 s.*



SOLUTION:

(a) (i) Distance travelled in the first 8 s is 30 m (see lines on graph).
(ii) Distance travelled in the interval $t = 8$ s to $t = 16$ s is 0 m (displacement remains constant at 30 m).

(b) Portion A B represents the greatest velocity (as the graph has the steepest slope in this portion).

(c) To determine velocity in the first 8 s:

$v_{av} = ?$ $\quad v_{av} = \frac{s}{t}$
$s = 30$ m $\quad = \frac{30}{8}$
$t = 8$ s $\quad = 3.75$ m s^{-1}

i.e. the velocity for the first 8 s is 3.75 m s^{-1}.

(d) To determine v_{av} for the 32 s:

$v_{av} = ?$ $\quad v_{av} = \frac{40}{32}$
$s = 40$ m
$t = 32$ s $\quad = 1.25$ m s^{-1}

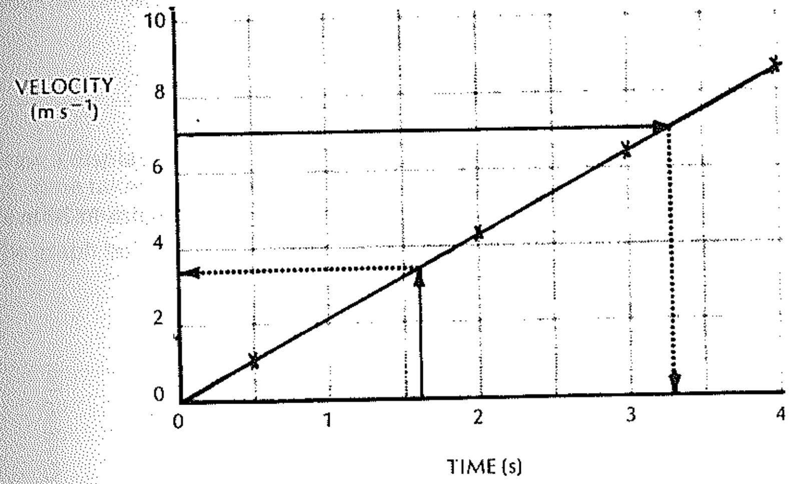
i.e. the average velocity for the 32 s is 1.25 m s^{-1}.

VELOCITY — TIME GRAPHS

TYPE EXAMPLE 19: A cyclist starting from rest has his velocity at various times recorded as follows. Construct a velocity — time graph and determine the:
(a) *time taken to reach a velocity of 7 m s^{-1}.*
(b) *velocity after 1.6 s.*

TIME (s)	0	1	2	3	4
VELOCITY (m s^{-1})	0	2.1	4.2	6.3	8.4

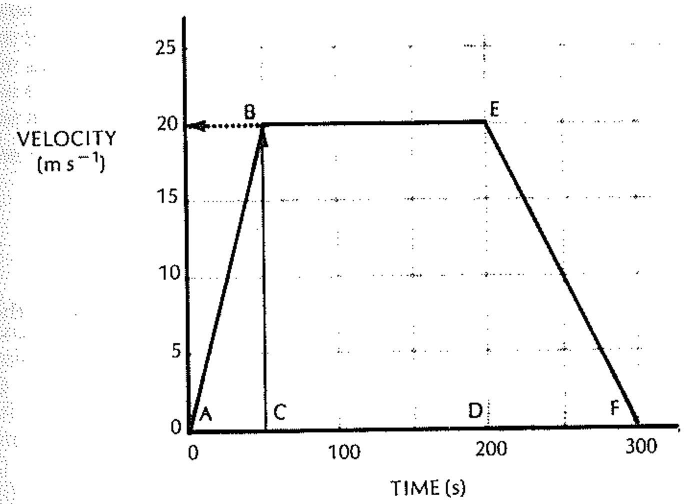
SOLUTION:



NOTE 1. A straight line velocity — time graph indicates a constant acceleration.
2. The slope of a velocity — time graph indicates the magnitude of the acceleration (the greater the slope, the greater the acceleration).
3. The area under a velocity — time graph represents the displacement.

From the graph using similar procedures to Type Example 17:
(a) a velocity of 7 m s^{-1} is attained after 3.3 s.
(b) after 1.6 s the cyclist's velocity is 3.4 m s^{-1}.

TYPE EXAMPLE 20: A velocity — time graph is shown below for a train travelling between two stations.



Use this graph to determine the:
(a) velocity of the train after 50 s.
(b) velocity of the train between 50 s and 200 s of the journey.
(c) acceleration of the train between t = 200 s and t = 300 s.
(d) total displacement of the train in 300 s.

SOLUTION:

(a) From the graph, the velocity after 50 s is 20 m s^{-1}.

(b) From the graph, velocity between t = 50 s and t = 200 s remains at 20 m s^{-1} (acceleration is zero as the graph has horizontal slope).

(c) Initial velocity, u (at t = 200s) = 20 m s^{-1}
Final velocity, v (at t = 300 s) = 0 m s^{-1}
Time interval, t = 300 s − 200 s = 100 s
Acceleration, a = ?

$$v = u + at$$
$$\therefore a = \frac{v - u}{t} = \frac{0 - 20}{100} = \frac{-20}{100} = -0.2 \text{ m s}^{-2}$$

i.e. the acceleration between t = 200 s and t = 300 s is −0.2 m s^{-2}.

(d) Displacement = area under a v − t graph
= area ABC + area BCDE + area DEF
= ½ (50 × 20) + (150 × 20) + ½ (100 × 20)
= 500 + 3 000 + 1 000
= 4 500 m

i.e. the total displacement is 4 500 m.

SET 14: GRAPHICAL REPRESENTATION OF MOTION

1. Observation of a motor boat provides the following data:

TIME (s)	1	2.5	5	8	10
DISPLACEMENT (m)	4.6	11.5	23	36.8	46

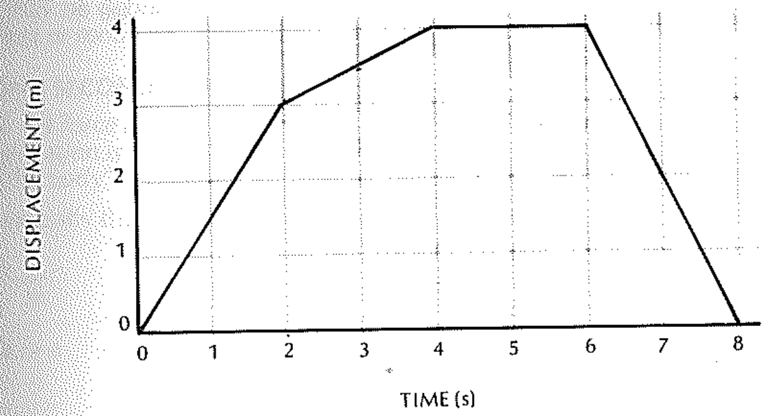
Draw a displacement — time graph to show this motion and use the graph to determine the:
(a) displacement after a
(i) 9 s period
(ii) 6.4 s period
(b) time interval to travel
(i) 15.3 m
(ii) 35 m.

2. Stroboscopic photography of a fast bowler delivering a ball down a cricket pitch gave the following results:

TIME (s)	0.2	0.4	0.6	0.8
DISPLACEMENT (m)	5	10	15	20

Draw a displacement — time graph to determine the:
(a) time taken to deliver the ball the length of the pitch, 20.1 m
(b) distance travelled by the ball in 0.35 s.

3. The graph below represents the motion of a toy electric car.



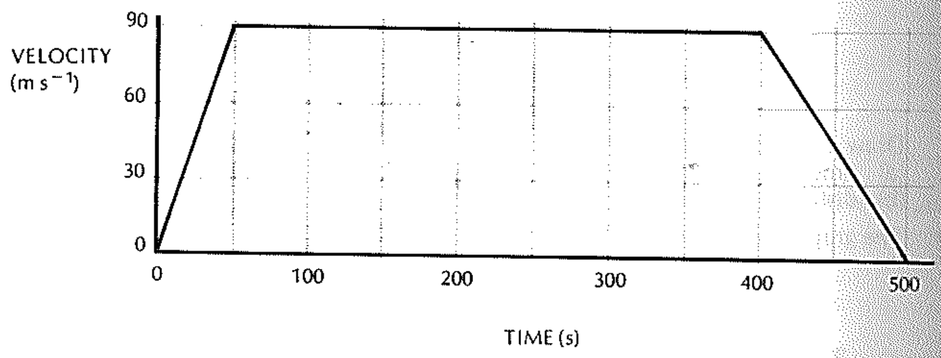
Use this graph to find:
(a) the velocity of the car —
(i) in the first 2 s
(ii) in the next 2 s (i.e. between t = 2 s and t = 4 s)
(b) in what time period the car was stationary
(c) the total displacement
(d) the average velocity for the 8 s
(e) the total *distance* travelled
(f) the average *speed* for the 8 s.

4. A stone falling from rest provided the following data:

TIME (s)	0	2	5	7	10
VELOCITY (m s^{-1})	0	19.6	49	68.6	98

Construct a velocity — time graph to estimate the:
(a) velocity of the stone after
(i) 6.5 s
(ii) 9 s
(b) time taken to reach a velocity of
(i) 17 m s^{-1}
(ii) 80 m s^{-1}

5. A short aeroplane journey is represented by the graph:



Use the graph to find the:

(a) velocity after 50 s
(b) acceleration during the first 50 s
(c) displacement while travelling at a constant velocity
(d) acceleration for the final 100 s of the journey.

CHAPTER 3: Newton's Laws of Motion

NEWTON'S FIRST LAW OF MOTION

"An object continues in its state of rest or uniform motion in a straight line unless acted on by a net external force".

This law is also known as the *law of inertia. Inertia is the tendency of a body to remain in its state of motion.*

The greater the mass of a body the greater is its inertia.

FORCE

A force may be defined as that which produces or tends to produce a change in the state of rest or motion of a body.

NEWTON'S SECOND LAW OF MOTION

"When a net force acts on a body the body is accelerated in the direction of the force. The acceleration is directly proportional to the force and inversely proportional to the mass of the body".

Mathematically this law can be expressed as:

$$a \propto \frac{F}{m}$$

or $F = k\,m\,a$ where
F = net force
k = proportionality constant
m = mass of the body
a = acceleration produced

When F, m and a are expressed in SI units $k = 1$ and the law may be written in its common form:

$$\boxed{F = m\,a}$$

UNITS OF FORCE

The unit of force is the *newton (N)* and is defined such that *a net force of 1 N acting on a mass of 1 kg produces an acceleration of 1 m s^{-2}.*

$$\begin{aligned} F &= m\,a \\ &= (1\text{ kg})(1\text{ m s}^{-2}) \\ \therefore 1\text{ N} &= 1\text{ kg m s}^{-2} \end{aligned}$$

TYPE EXAMPLE 21: What force acting on a mass of 8 kg will produce an acceleration of 1.5 m s^{-2}?

F = ?
m = 8 kg
a = 1.5 m s^{-2}

$$\begin{aligned} F &= ma \\ &= 8 \times 1.5 \\ &= 12\text{ N} \end{aligned}$$

i.e. a force of 12 N is required (in the direction of the acceleration).

TYPE EXAMPLE 22: A force of 50 N is applied to a mass of 12.5 kg. What is the acceleration produced?

$F = 50$ N
$m = 12.5$ kg
$a = ?$

$$F = ma$$
$$\therefore a = \frac{F}{m} = \frac{50}{12.5} = 4 \text{ m s}^{-2}$$

i.e. an acceleration of 4 m s^{-2} is produced (in the direction of the force).

SET 15: NEWTON'S SECOND LAW OF MOTION I

1. A mass of 6 kg is accelerated at 3.5 m s^{-2}. What net force is applied?
2. Determine the magnitude of the force which when acting on a mass of 2.5 kg produces an acceleration of 5 m s^{-2}.
3. What force must act on a car of mass 2 000 kg to produce an acceleration of 1.6 m s^{-2}?
4. Find the acceleration produced when a force of 200 N acts on a body of mass 40 kg.
5. A force of 36 N acts on a 24 kg mass. What acceleration is produced?
6. What acceleration is produced when a net force of 16 N acts on a mass of 48 kg?
7. Determine the acceleration resulting when a force of 625 N acts on a body of mass 125 kg?
8. A force of 72 N produces in a body an acceleration of 1.2 m s^{-2}. What is the mass of the body?
9. What is the mass of a car which is accelerated at 2 m s^{-2} when acted on by a force of 2 800 N?
10. A force of 3 N is applied to a block on a smooth horizontal table and produces an acceleration of 11 m s^{-2}. What is the mass of the block?

TYPE EXAMPLE 23: A force of 25 N acts on a 10 kg body at rest. Determine after a period of 3 s:
(a) the velocity.
(b) the displacement.

(a) $F = 25$ N
$m = 10$ kg
$u = 0$
$a = ?$
$v = ?$
$t = 3$ s

$$F = ma$$
$$\therefore a = \frac{F}{m} = \frac{25}{10} = 2.5 \text{ m s}^{-2}$$
$$v = u + at = 0 + (2.5 \times 3) = 7.5 \text{ m s}^{-1}$$

i.e. the velocity is 7.5 m s^{-1} (in the direction of the force).

(b) $u = 0$
$a = 2.5$ m s^{-2} (part a)
$t = 3$ s
$s = ?$

$$s = ut + \tfrac{1}{2} at^2 = 0 + \tfrac{1}{2}(2.5 \times 3^2) = \tfrac{1}{2}(2.5 \times 9) = 11.25 \text{ m}$$

i.e. the displacement is 11.25 m (in the direction of the force).

TYPE EXAMPLE 24: What force is required to slow down a body of mass 50 kg from 20 m s^{-1} to 12 m s^{-1} in 4 s?

$F = ?$
$m = 50$ kg
$u = 20$ m s^{-1}
$v = 12$ m s^{-1}
$t = 4$ s

$$v = u + at$$
$$\therefore a = \frac{v-u}{t} = \frac{12-20}{4} = \frac{-8}{4} = -2 \text{ m s}^{-2}$$
$$F = ma = 50 \times (-2) = -100 \text{ N}$$

i.e. a force of 100 N in the opposite direction to the motion is required.

SET 16: NEWTON'S SECOND LAW OF MOTION II

1. A force of 60 N acts on a mass of 30 kg initially at rest. What is the velocity after a period of 8 s?
2. A stationary mass of 12 kg is acted on by a force of 15 N for 6 s. Determine the:
   (a) final velocity
   (b) resulting displacement.
3. A body of mass 6.5 kg initially moving at 7.5 m s^{-1} is acted on by an accelerating force of 19.5 N for 1.5 s. Calculate the:
   (a) final velocity
   (b) displacement.
4. Determine the force which will accelerate a body of mass 4.5 kg from rest to 7.5 m s^{-1} in 5 s.
5. A vehicle of mass 1 250 kg is accelerated from 6.5 m s^{-1} to 19.5 m s^{-1}. Determine the force which is required to produce this change in 2 s.
6. What force acting on a body of mass 16 kg initially at rest, will produce a displacement of 875 m in 5 s?

7. A mass of 9 kg at rest is acted on by a force of 16.2 N. What period of time must elapse for this mass to be moved 90 m?
8. What force is required to bring to rest in 3 s a 1 000 kg body moving at 15 m s^{-1}?
9. A mass of 28 kg is brought to rest by a force of 98 N in 13 s. What is the initial velocity of the mass?
10. A block of wood sliding down an incline is brought to rest in 2 s by a retarding force of 0.75 N in a distance of 1 m. Find the:
    (a) average velocity
    (b) initial velocity
    (c) acceleration
    (d) mass of the block.

WEIGHT AND MASS

Mass refers to the quantity of matter in a body and is measured in kilograms (kg).

Weight is the earth's gravitational force of attraction on a body and is directed towards the centre of the earth. As weight is a force it is measured in *newtons (N).*

Freely falling bodies are accelerated at 10 m s^{-2} (g) near the earth's surface.

A mass of 1 kg falling freely near the surface of the earth will be accelerated at 10 m s^{-2}. The gravitational force of attraction producing this acceleration can be calculated by:

$$\begin{aligned} F &= ma \\ &= (1\ kg)\ (10\ m\ s^{-2}) \\ &= 10\ kg\ m\ s^{-2} \\ &= 10\ N \end{aligned}$$

From this it can be seen that the gravitational force of attraction on a 1 kg mass is 10 N and from the definition of weight, the weight of a mass of 1 kg is 10 N downward.

As all bodies are accelerated at g near the earth's surface the weight of a body can be determined from:

$$F = ma$$

$$\boxed{F_w = mg}$$

where: F_w = weight (N)
m = mass (kg)
g = acceleration due to gravity

TYPE EXAMPLE 25: Determine the weight of a 6 kg mass.

$m = 6$ kg
$g = 10$ m s^{-2}
$F_w = ?$

$$\begin{aligned} F_w &= mg \\ &= 6 \times 10 \\ &= 60\ N \end{aligned}$$

i.e. the weight of a 6 kg mass is 60 N downward.

SET 17: WEIGHT

Use g = 10 m s^{-2} for acceleration due to gravity near the earth's surface.

1. What is the weight of a:
   (a) 52 kg mass? (b) 2.6 kg mass?
2. Determine the mass of bodies with the weights:
   (a) 350 N (b) 5 N
3. Find the weight of a student of mass 48 kg.
4. What is the weight of a 1 000 kg motor car?
5. What is the mass of a crate which weighs 1 200 N?
6. A vehicle of mass 800 kg has a horizontal force equal to its weight applied to i
   Find the:
   (a) weight of the vehicle
   (b) acceleration produced.
7. A body of mass 4 kg at rest is acted upon by a force equal to half its weigh
   Determine after 3 s, the:
   (a) velocity
   (b) displacement.
8. A body is suspended on a spring balance and shows a weight of 20 N. This sam balance and body on the surface of the moon would show 3.2 N. What is the a celeration due to gravity on the surface of the moon?

NEWTON'S THIRD LAW OF MOTION

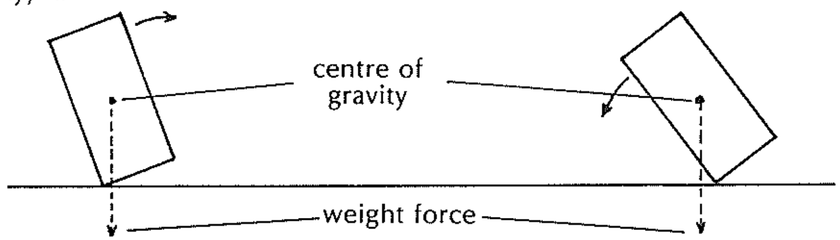
"Whenever a body exerts a force on another body this second body exerts a forc equal in magnitude but opposite in direction on the first."

Forces always occur in pairs. Each of these forces acts on a different body. For exan ple, when a book rests on a table the weight of the book acts on the table and tł table exerts an equal force (but in the opposite direction) on the book.

CENTRE OF GRAVITY

The entire weight of an object can be considered to act through one point. Thi point is called the *centre of gravity*. The position of the centre of gravity i determined by the object's shape and its distribution of mass.

Objects fall over if the weight force (i.e. the vertical line through the centre o gravity) falls outside the base.



Therefore an object's stability is increased by:
(i) lowering the centre of gravity *and*
(ii) broadening the base

CHAPTER 4: Vectors

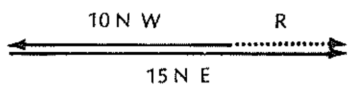
DETERMINATION OF RESULTANTS FOR VELOCITY, FORCE AND ACCELERATION VECTORS.

The *"head to tail"* or *"triangle"* method for adding two or more successive displacement vectors has already been explained in Chapter 2. The same method can be used to add two or more velocity, force or acceleration vectors. Indeed if there are more than two of these vectors or the two act in the same line then the "head to tail" method must be used.

TYPE EXAMPLE 26: Two men pull on a trolley from opposite sides, one with a force of 10 N west and the other with a force of 15 N east. What is the resultant force on the trolley?

Sketch vectors 'head to tail' (approximately to scale).

Direction: N



A graphical solution is not required and the calculation of the resultant is obvious.

Resultant, R = (15 N) − (10 N) = 5 N east

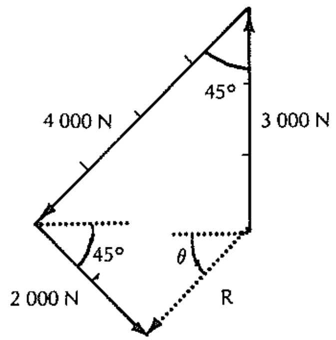
i.e. the resultant force is 5 N east.

TYPE EXAMPLE 27: Three tractors are connected by chains to the same point. The force exerted by each is 3 000 N north, 4 000 N south-west and 2 000 N south-east respectively. What is the resultant force at that point?

A graphical solution must be used.

Direction: N

Scale: 1 cm = 1 000 N



Length of resultant, R = 1.9 cm
Magnitude of R = 1.9 × 1 000
= 1 900 N
Direction of resultant = W 44° S (from angle θ)

i.e. the resultant force is 1 900 N W 44° S.

Often two velocity, force or acceleration vectors act simultaneously at an angle at the same point. Then it may seem more logical to consider the *"parallelogram"* method for adding these two vectors:

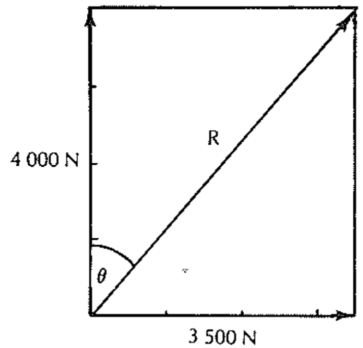
1. Use arrows to represent the magnitude and direction of the two vectors acting at one point.
2. Complete the parallelogram for which these two vectors are adjacent sides.
3. Draw in the resultant which is the diagonal of the parallelogram from the point of application of the two vectors.
4. If the vectors have been drawn to scale then the magnitude and direction of the resultant can be measured — otherwise both can be calculated.

TYPE EXAMPLE 28: Two tow trucks exert forces of 4 000 N north and 3 500 N east on a bogged car. What is the resultant force acting on the bogged car?

A. GRAPHICAL SOLUTION

Direction: N

Scale: 1 cm = 1 000 N

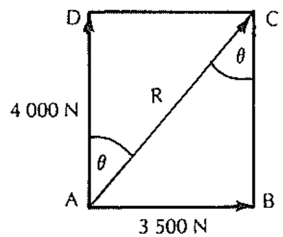


Length of resultant, R = 5.3 cm
Magnitude of R = 5.3 × 1 000
= 5 300 N
Direction of resultant = N 41° E

i.e. the resultant force is 5 300 N N 41° E.

B. CALCULATION SOLUTION (for vectors at right angles only)

1. Sketch the vector parallelogram (approximately to scale).



2. Using Pythagorus' Theorem calculate a value for the magnitude of the resultant, AC.

$$\begin{aligned} AC &= \text{Resultant} \\ &= ? \\ AB &= 3\,500 \text{ N} \\ BC &= AD \\ &= 4\,000 \text{ N} \end{aligned} \qquad \begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= (3\,500)^2 + (4\,000)^2 \\ &= 28\,250\,000 \\ \therefore AC &= 5\,315 \text{ N} \end{aligned}$$

3. Using Trigonometric ratios determine the direction (θ) of the resultant.

$$\begin{aligned} \text{Tan } \theta &= \frac{AB}{BC} \\ &= \frac{3\,500}{4\,000} \\ &= 0.875 \\ \therefore \theta &= 41.2° \end{aligned}$$

i.e. the resultant force is 5 315 N N 41.2° E.

It can be seen that triangle A B C within the parallelogram A B C D corresponds to that obtained using the *"head to tail"* or *"triangle"* method. Thus, of-course, both methods give the *same* resultant and *either* method can be used.

SET 18: RESULTANT VECTORS I

NOTE: The following problems are to be solved using the "head to tail" method.

1. A boat heads upstream directly against a current flowing at 4 m s^{-1} SE. If the velocity of the boat without the current would be 20 m s^{-1} NW, what is the resultant velocity of the boat?
2. Find the resultant acceleration acting on an object suspended on a string experiencing an acceleration of 12.5 m s^{-2} upwards due to tension in the string and an acceleration of 10 m s^{-2} downwards due to its weight.
3. Two men pull on a stump with forces of 250 N and 300 N, both NE. Determine the resultant force acting on the stump.
4. Two tug boats push on opposite sides of a ship with forces of 20 000 N. What is the net force acting on the ship if one pushes in a northerly direction and the other in a southerly direction?
5. Three horses are roped to a stake. The forces they exert are 750 N south-west, 360 N north-east and 570 N south respectively. What is the resultant force acting on the stake?
6. A lift is accelerating downwards. The forces acting on it are a weight of 2 000 N downward and a force of tension in the supporting cable of 1 760 N upward. Find the resultant force acting on the lift.
7. Forces of 70 N east, 55 N west 30° north, 60 N south and 40 N east 60° south act simultaneously on the same point. Determine the resultant force acting at this point.
8. Three forces of 150 N act simultaneously at the same point. If the angle between any two of these forces is 120°, what is the resultant force at the point of application of the forces?
9. The velocity of a plane is 200 m s^{-1} west without any wind. If there is a tail breeze at 32 m s^{-1} west, determine the resultant velocity of the plane.
10. Determine the resultant velocity of the plane in question 9 if there is a head wind of 27 m s^{-1} east.

SET 19: RESULTANT VECTORS II

Graphically determine the resultant for each of the following:

1. Two men exert forces of 500 N east and 450 N south-east respectively on a donkey which is refusing to move. What is the resultant force acting on the donkey?
2. A plane heads due east at 180 m s^{-1} with a wind from the south-west of 30 m s^{-1}. Determine the resultant velocity of the plane.
3. A ship heads due north at 40 km h^{-1}. A current of 5 km h^{-1} from the north-west causes it to be off course. Find the resultant velocity of the ship.
4. Two forces acting simultaneously on a crate produce accelerations of 5 m s^{-2} N 30° W and 3 m s^{-2} N 70° W. What is the resultant acceleration of the crate?
5. Two bulldozers use chains to pull on a rock. One exerts a force of 3 000 N while the other exerts a force of 2 700 N. If the angle between the two chains is 60°, find the resultant force acting on the rock. (For direction give the angle between the resultant and the larger force).

SET 20: RESULTANT VECTORS III

Using Pythagorus' Theorem and trigonometric ratios determine the resultant in each of the following:

1. A plane's velocity through the air is 100 m s^{-1} west. There is a wind from the south at 15 m s^{-1}. Find the resultant velocity of the plane.
2. A motor boat heads across a river at 15 m s^{-1}. If there is a current of 5 m s^{-1} at right angles to the direction of travel find the resultant velocity of the boat. (For direction give the angle between the resultant and the original direction.)
3. A train is moving at 5 m s^{-1} east and a passenger jumps off at 5 m s^{-1} south. Find the passenger's resultant velocity.
4. Two tow trucks pulling on a bogged vehicle exert forces of 2 000 N and 1 500 N at right angles to one another. What is the resultant force? (For direction give the angle between the resultant and the larger force.)
5. Forces of 160 N west and 220 N south act simultaneously at a point. What is the resultant force?

NOTE: When the resultant of the forces acting on an object is zero, the object is in *EQUILIBRIUM* i.e. it remains at rest or at uniform velocity.

$$\begin{aligned}
\text{length of VI} &= 2 \text{ cm} \\
\therefore \text{ The distance between the vertex} & \\
\text{and the image} &= (2 \times 3) \text{ cm} \\
&= 6 \text{ cm} \\
\text{length of IM} &= 0.67 \text{ cm} \\
\therefore \text{ height of image} &= (0.67 \times 3) \text{ cm} \\
&= 2 \text{ cm}
\end{aligned}$$

i.e. The image is virtual, upright and diminished. It is 6 cm from the vertex on the opposite side to the object and 2 cm high.

For a convex mirror the image is *always* virtual, upright and diminished.

SET 42: CURVED MIRRORS

In each case find the nature, location and height of the image by using the rules of construction.

1. An object 15 cm high is 75 cm from a concave mirror of focal length 25 cm.
2. The tail of an object 10 cm high sits on the centre of curvature of a concave mirror of focal length 20 cm. In this case use a ray from the head of the object through the principal focal point and reflected parallel to the principal axis as the second ray needed to construct the diagram.
3. A candle 2 cm high is 9 cm from a concave mirror of focal length 5 cm.
4. The tail of an object 5 cm high sits on the principal focal point of a concave mirror of focal length 10 cm.
5. A dentist holds a concave mirror of focal length 3 cm a distance of 1 cm from a tooth which is 1 cm high.
6. Rays from the same point on a very distant object may be considered parallel if they reach a mirror. Describe the image formed by a reflecting telescope using a concave mirror of focal length 200 cm for a distant star.
7. A car 1.5 m high is 10 m behind a car in which the rear vision mirror is a convex mirror of focal length 0.5 m.
8. An object 10 cm high is 10 cm from a convex mirror of focal length 10 cm.
9. A man views his head, which is 20 cm high, from 50 cm in front of large shaving mirror which is a concave mirror of focal length 100 cm.
10. If an object 10 cm high is 100 cm from a concave mirror of focal length 20 cm, describe the image and determine the magnification using the definition given in question 10 Set 36.

ANSWERS

SET 1

1. 6 200 m **2.** 186 s **3.** 9 000 s **4.** 1.76 m **5.** 5.5 kg
6. 0.025 kg **7.** 1 500 kg **8.** 0.018 m **9.** 0.075 m^2 **10.** 0.001 m^3

SET 3

1. 216 m S **2.** 47 m downfield **3.** 13 m S **4.** 115 km SW **5.** 23 m down

SET 4

1. 280 m S 15° W **2.** 560 km W 63° S **3.** 45 cm 27° to the horizontal
4. 77 cm 19° to the horizontal **5.** 17.1 km N 52° W

SET 5

1. 424 km NW **2.** 10 m 53° to the vertical **3.** 1 300 m E 67° S
4. 20 m 30° to the horizontal **5.** 2 km N 8° E

SET 6

1. 8 $m\ s^{-1}$ **2.** 25 $m\ s^{-1}$ **3.** 9 $m\ s^{-1}$ **4.** 150 m **5.** 30 m
6. 324 000 m **7.** 14 s **8.** 3 s **9.** 60 s **10.** 8.5 h
11. 20 km **12.** 480 $km\ h^{-1}$

SET 7

1. (a) 8.8 $m\ s^{-1}$ **(b)** 5.6 $m\ s^{-1}$ W **(c)** 0 **2. (a)** 350 s **(b)** 5 km N 53° W
(c) 14.3 $m\ s^{-1}$ N 53° W **3. (a)** 30 m **(b)** 1.5 $m\ s^{-1}$ **4. (a)** 70 s **(b)** 5 $m\ s^{-1}$
5. 3 000 m S **6.** 2 m **7.** 2 $m\ s^{-1}$ **8.** 50 s **9.** 800 m **10.** 14 $m\ s^{-1}$

SET 8

1. 1.8 $m\ s^{-1}$ **2.** 24 $m\ s^{-1}$ **3.** 32.4 $m\ s^{-1}$ **4.** 21 $m\ s^{-1}$ **5.** 2.5 $m\ s^{-2}$
6. 3 $m\ s^{-2}$ **7.** 3 $m\ s^{-2}$ **8.** 2 s **9.** 20 s **10.** 6 $m\ s^{-2}$

SET 9

1. 3 $m\ s^{-2}$ **2.** −10 $m\ s^{-2}$ **3. (a)** 27 $m\ s^{-1}$ **(b)** 24 s **4.** 110 $m\ s^{-1}$
5. 6 s **6.** 10 s **7.** 6 s **8.** 10 $m\ s^{-1}$

SET 10

1. (a) 256 m **(b)** 32 $m\ s^{-1}$ **(c)** 44 $m\ s^{-1}$ **2.** 14.4 m
3. (a) 30 m **(b)** 8 $m\ s^{-1}$ **(c)** 5 $m\ s^{-1}$ **4. (a)** 500 m **(b)** 20 $m\ s^{-1}$ **(c)** 10 $m\ s^{-1}$
5. 46 m **6. (a)** 10 m **(b)** 10 $m\ s^{-1}$ **(c)** 5 $m\ s^{-1}$ **7. (a)** 25.5 m **(b)** 0.5 $m\ s^{-1}$
8. (a) 120 m **(b)** 30 $m\ s^{-1}$ **(c)** 10 $m\ s^{-1}$ **9. (a)** 18 s **(b)** 648 m
10. (a) 90 m **(b)** 30 $m\ s^{-1}$

SET 11

1. (a) 96 $m\ s^{-1}$ **(b)** 576 m **(c)** 48 $m\ s^{-1}$ **2. (a)** 30.5 $m\ s^{-1}$ **(b)** 1.5 $m\ s^{-2}$ **(c)** 549 m
3. 6 s **4.** 25 s **5. (a)** −6 $m\ s^{-2}$ **(b)** 147 m **6. (a)** 4.8 $m\ s^{-2}$ **(b)** 4.8 $m\ s^{-1}$
(c) 9.6 $m\ s^{-1}$ **7.** 6.75 m **8.** 4.8 m **9.** 200 m **10.** 272 m

SET 12

1. (a) 40 $m\ s^{-1}$ **(b)** 80 m **2. (a)** 12 $m\ s^{-1}$ **(b)** 7.2 m **3.** 31.25 m
4. 4.05 m **5.** 75 m **6. (a)** 20 m **(b)** 45 m **(c)** 25 m

SET 13

1. **(a)** 35 m s^{-1} **(b)** 56.25 m **2.** **(a)** 25 m s^{-1} **(b)** 20 m
3. **(a)** 400 m s^{-1} upwards **(b)** 400 m s^{-1} downwards
4. 3 m s^{-1} downwards **5.** **(a)** 60 s **(b)** 18 000 m **6.** **(a)** 1.6 s **(b)** 12.8 m
7. 3 s **8.** 4 s **9.** **(a)** 1.6 s **(b)** 16 m s^{-1} **10.** 3.4 s

SET 14

1. **(a)** **(i)** 41.4 m **(ii)** 29.4 m **(b)** **(i)** 3.3 s **(ii)** 7.6 s **2.** **(a)** 0.8 s **(b)** 8.75 m
3. **(a)** **(i)** 1.5 m s^{-1} **(ii)** 0.5 m s^{-1} **(b)** 4 s to 6 s **(c)** 0 **(d)** 0 **(e)** 8 m **(f)** 1 m s^{-1}
4. **(a)** **(i)** 63.7 m s^{-1} **(ii)** 88.2 m s^{-1} **(b)** **(i)** 1.7 s **(ii)** 8.2 s
5. **(a)** 90 m s^{-1} **(b)** 1.8 m s^{-2} **(c)** 31 500 m **(d)** −0.9 m s^{-2}

SET 15

1. 21 N **2.** 12.5 N **3.** 3 200 N **4.** 5 m s^{-2} **5.** 1.5 m s^{-2}
6. 0.33 m s^{-2} 7. 5 m s^{-2} **8.** 60 kg **9.** 1 400 kg **10.** 0.27 kg

SET 16

1. 16 m s^{-1} **2.** **(a)** 7.5 m s^{-1} **(b)** 22.5 m **3.** **(a)** 12 m s^{-1} **(b)** 14.63 m
4. 6.75 N **5.** 8 125 N **6.** 1 120 N 7. 10 s **8.** −5 000 N
9. 45.5 m s^{-1} **10.** **(a)** 0.5 m s^{-1} **(b)** 1 m s^{-1} **(c)** −0.5 m s^{-2} **(d)** 1.5 kg

SET 17

1. **(a)** 520 N **(b)** 26 N **2.** **(a)** 35 kg **(b)** 0.5 kg **3.** 480 N **4.** 10 000 N
5. 120 kg **6.** **(a)** 8 000 N **(b)** 10 m s^{-2} 7. **(a)** 15 m s^{-1} **(b)** 22.5 m
8. 1.6 m s^{-2}

SET 18

1. 16 m s^{-1} NW **2.** 2.5 m s^{-2}upwards **3.** 550 N NE **4.** 0
5. 890 N S 18° W **6.** 240 N downwards **7.** 79.5 N E 58° S **8.** 0
9. 232 m s^{-1} W **10.** 173 m s^{-1} W

SET 19

1. 878 N E 21° S **2.** 202 m s^{-1} E 6° N **3.** 37 m s^{-1} N 6° E
4. 7.5 m s^{-2} NW **5.** 4 940 N at 28°

SET 20

1. 101 m s^{-1} W 9° N **2.** 15.8 m s^{-1} at 18° **3.** 7.1 m s^{-1} SE
4. 2 500 N at 37° **5.** 272 N W 54° S

SET 21

1. 540 J **2.** 55 000 J **3.** 5 m **4.** 4 000 N **5.** 450 J
6. 150 000 J **7.** 3.3 kg **8.** 240 000 J **9.** 3.6 m
10. **(a)** 6 J **(b)** 3.6 J **11.** 18 000 J **12.** 2 000 kg

SET 22

1. 1 280 J **2.** 1 000 J **3.** 2 N **4.** 64 000 J **5.** 30 m

SET 23
1. 625 J **2.** 62.5 J **3.** 16 kg **4.** 100 m s^{-1} **5.** 18 J
6. 75 000 J 7. 20 000 J **8.** 18 000 J **9.** 32 000 J **10.** 6 400 J

SET 24
1. 125 000 J **2.** 800 J **3.** 1 200 J **4.** 50 m **5.** 1.5 J
6. 45 m 7. 9.2 m **8.** 0.4 kg **9.** 0.032 m **10.** **(a)** 5 J **(b)** 40 J

SET 25
1. 4 W **2.** **(a)** 1.5 kJ **(b)** 50 s **3.** 4 800 kJ **4.** 200 W
5. (a) 5 s **(b)** 50 000 J **(c)** 10 000 W **6.** 4 000 W 7. 50 000 W
8. 2 400 kW **9.** 833 m **10.** 250 m

SET 26
1. 800 kg m^{-3} **2.** (a) 2.7 g cm^{-3} (b) 2.7 x 10^3 kg m^{-3} **3.** 7 860 kg m^{-3}
4. 19.3 g cm^{-3} **5.** 1 200 kg **6.** 93 600 kg **7.** 0.25 m^3
8. 4.5 m^3 **9.** 305 kg **10.** 53.5 g

SET 27
1. 1 000 Pa **2.** 10 Pa **3.** 50 Pa **4.** 0.014 Pa
5. 3 000 000 Pa **6.** 18 889 Pa, 2 833 Pa **7.** 140 000 N
8. 0.25 m^2 **9.** 325 000 N **10.** 0.003 m^2

SET 28
1. 15 000 Pa **2.** 2 800 Pa **3.** 14 000 Pa **4.** 0.4 m
5. 1 200 kg m^{-3} **6.** 5 500 000 Pa **7.** 1.4 m **8.** liquid A
9. 7.5 m **10.** (a) 1.25 kPa (b) 2.00 cm

SET 29
1. 12 L **2.** 380 mm Hg **3.** 500 kPa **4.** 9.4 mL **5.** 1600 L
6. 6.5 L 7. 3.75 atm **8.** 7 m^3 **9.** 250 mL **10.** volume × 4

SET 30
1. 423 K **2.** 300 K **3.** 546 K **4.** 190 K **5.** 168 K
6. 4°C 7. 273°C **8.** 17°C **9.** −63°C **10.** −269°C

SET 31
1. 15 L **2.** 67 K **3.** 60 mL **4.** 95.5°C **5.** −185.5°C
6. 10.3 m^3 7. 75 L **8.** −146°C **9.** 6 L **10.** 293°C

SET 32
1. 292 kPa **2.** −124°C **3.** 528 kPa **4.** 55 cm Hg **5.** 4.8 atm
6. 10°C (inc) 7. 4 641°C **8.** pressure × 2 **9.** temp × ½ **10.** −213°C

SET 33
1. 64 kPa **2.** 1.37 L **3.** 2.38 L **4.** 1 atm **5.** 1 485°C
6. −23°C 7. 99°C **8.** 93 L **9.** 280 K **10.** 11.1 mL

SET 34
1. 12 V **2.** 12 V **3.** 960 ohm **4.** 6 A
5. 0.15 A **6.** 0.0014 V **7.** 83.3 ohm **8.** 0.0025 A
9. 2.1 A **10.** (a) 2.45 A (b) 0.43 V